Extreme Value Theory can save your neck

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Abstract

How tall should one design an embankment so that the sea reaches this level only once in 100 years? How large might a possible stock market crash be tomorrow?

Many real life questions require estimation, but since no data or only few has been observed – as by definition extreme events are rare – essential estimations are more often based on feeling than on fact. Extreme Value Theory (EVT) is a branch of statistics that deals with such rare situations and that gives a scientific alternative to pure guesswork. In this white paper we show how Extreme Value Theory can literally save your neck.

1 The unknown kills you

Frank Clancy's famous words "the usual risks of the desert are: rattlesnakes, the heat, and lack of water" tell us what risks we have to cope with on our next trip in the desert. Now we can prepare accordingly.

Difficulty starts upon facing the unknown.

Might EVT have saved the space shuttle Challenger? The explosion of the space shuttle Challenger on January 28, 1983 is important with respect to risk management as it was the consequence of an extreme event: the exceptionally low temperature (15 degrees F lower than the next coldest previous launch) the night before launching ultimatively led to failure of the O-rings which caused the disaster. Using standard EVT-analysis, one could have spotted that one should not launch at such cold temperature, despite having no measurements at such low temperatutre.

2 What are Extremes? Why should I care?

Extremes are unusual or rare events. In classical data analysis tasks extremes are often labelled as outliers and even ignored. This means the data gets raped to fi t the model.

If you only seek estimations about everyday events, it might not matter if you cut off extreme data, but if you ask questions about events that do not happen very often, you should apply Extreme Value Theory; especially as these are the situations where you have the most to lose or to win: stockmarket crash, war, natural disasters, ...

For the layman earthquakes, hurricanes, and stock market crashes are surprising phenomena following no rule, but careful analysis has helped to discover distributions that acceptably model these extreme events. If you measure the size of many people and plot the heights in a histogram, you will discover a mathematical rule, likely the well-known bell-shaped curve of a normal distribution. Amazingly much of real-life data follow the normal distribution and its cousins, which are well understood and easily computable. However, when looking for extremes, which are in the tails of distributions, you often find that in real life situations these tails are fatter (more heavy) than classical distributions predict (cf. Figure 1).

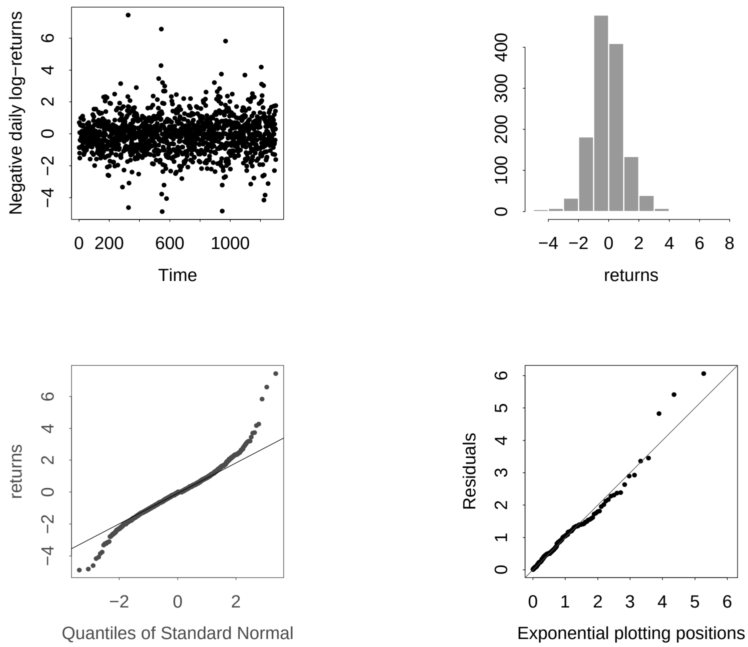

Figure 1: **Top left:** Daily negative log returns of the Dow Jones index (DJI). Financial mathematics normally works with log returns as the distribution can be modeled easier than with the raw data. **Top right:** A histogram of the returns. **Bottom left:** A normal distribution on the daily negative log returns does not fit well on the tails of the distribution. **Bottom right:** The tails are modeled using basic EVT, which works well, though 3 outliers (crashes . . .) remain, which require more sophisticated modelling.

Emil Julius Gumbel, a German mathematician, pacifist and anti-Nazi campaigner developed new distributions in the 1950s. The Gumbel-distribution, the Generalized Extreme Value distribution and the Generalized Pareto Distribution (GPD) are just the tip of the ice-berg of an entire new and quickly growing branch of statistics. The first application was to answer environmental questions, quickly followed by the finance industry.

3 Some have seen the light . . .

A large group of people from various domains have understood the need for EVT. Each of their specific problem domains yields heavy-tailed data.

Insurance, especially re-insurance companies, need to predict the amounts of large insurance losses. Rapid climate changes make this task urgent.

For the first time in the history of our planet, mankind is about to change the climate significantly and possibly irreversibly, without having any idea of the consequences that will have. ... With economic and insured losses increasing in volume by a factor of 3 and 5 respectively since the 1960s, we definitely have a trend which, without exaggeration, may be regarded as dramatic. (Munich Re, 1990)

Extreme weather leads to extreme cost. The summer 2002 flooding in Europe is estimated to have cost insurance companies about $2.5 bn, which is just peanuts compared to the Kobe earthquake, which is estimated to have cost insurance companies between $100bn and $150bn.

EVT is most recently used in the finance industry. Since its introduction to the Basle committee (Bank of International Settlements, BIS) in the early nineties, Value-at-Risk[1] (VaR) has become the standard risk measurement to protect portfolio holders against adverse market conditions and prevent them from taking extraordinary risks. Existing standard methods to calculate the VaR assume normality of the data. Unfortunately this assumption is known to be often strongly violated, as the unconditional distribution of financial time series is known to be heavy-tailed. This gave birth to the use of EVT methods to model the tail and to estimate VaR more reliably (see Figure 2 on the next page).

Long Term Capital Management (LTCM), a highly regarded hedge fund, nearly collapsed in September 1998. John Meriweather and the nobel laureates Robert Merton and Myron Scholes lead a big hedge-fund with massive leverage: with $4.8 bn, they controlled $160 bn and they were betting on the convergence of different securities. Alan Greenspan and many investment banks saved the fund during a major liquidity crises, as Greenspan was afraid that the shock waves might cause a market crash.

LTCM's worst case scenario estimated only a 20% loss instead of the 60% they had once things started to go wrong. EVT might have helped hem.

In an interview with the Wall Street Journal (August 21, 2000) John Meriweather said: *With globalisation increasing, you'll see more crises. Our whole focus is on extremes now – what's the worst that can happen to you in any situation – because we never want to go through that again.* This disgrace, without any doubt, did affect the full power of mathematics in finance and in an interview with the Swiss Newspaper Le Temps (December 1st, 2003), Prof. Paul Embrechts added: *The real price observations didn't stop contradicting the Black&Scholes formulas. The statement that the volatility is constant is compromised. Constant? It most likely follows the state of heart of the traders! . . . Mathematics must correct these formulas to suit the reality.*

Professor Embrechts, called Mister *Extreme Values*, found a natural audience in the financial world. However, internet traffic, structural reliability or biotech analyses are other prime targets for EVT applications, as their data distributions display

[1] VaR is formally defined as the α-quantile of the Profit-and-Loss (P&L) distribution of a portfolio of value V_t at time t over the holding period, or horizon, h. Alternative measures of market risk have been proposed such as Expected Shortfall (ES) which measures the expected loss exceeding the VaR.

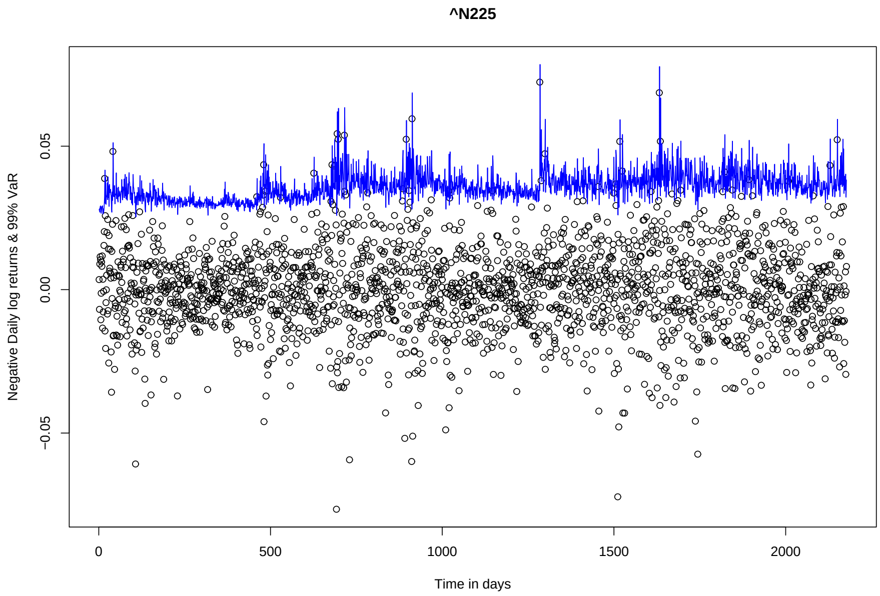

Figure 2: The points are the negative daily log returns of the Nikkei 225 (from Jan 1st, 1997 to Jan 1st, 2001) and the top-line is the estimated 99% VaR against time (days). This approach of VaR estimation is an extension of a classical EVT model and allows realistic models. A backtesting study would show how correctly the method estimates the VaR.

heavy tails, too. Even corporate companies should investigate possible uses of EVT, as their risk managers have to worry about the risks of low-probability events that could lead to catastrophic losses.

The potential of extreme value theory applied to financial problems has only been recognized recently. The end of the last decade has been characterized by significant instabilities in financial markets worldwide. This has led to numerous criticisms about the existing systems for risk management and motivated the search for more appropriate methodologies able to cope with rare events of heavy consequences.

The typical question one would like to answer is: "If things go wrong, how wrong can they go?" So the problem is to model these rare phenomena, which mainly lie outside the range of available observations. In such situations it is essential to rely on a well founded methodology. EVT provides a confirmed theoretical foundation on which we can build statistical models describing extreme events.

4 No silver bullet, but better than nothing

EVT is the most scientific approach to a difficult problem: predicting the size of a rare event.

> *There is always going to be an element of doubt, as one is extrapolating into into areas one doesn't know about. But what EVT is doing is making the best use of whatever data you have about extreme phenomena.*
> *– Richard Smith*

EVT will become more wide spread as a technique that risk managers will use more often. But successful Risk Management can only be integrated Risk Management, as if one does not look at all risk factors together, one loses portfolio effects and maybe solves the wrong problem. EVT research has shown ways how to combine different risk factors.

> *The key message is that EVT cannot do magic – but it can do a whole lot better than empirical curve-fitting and guesswork. My answer to the sceptics is that if people aren't given well-founded methods like EVT, they'll just use dubious ones instead. – Jonathan Tawn*

If you look at fat tails, consider using EVT, as EVT is too expensive to ignore. EVT as well as any other model is only an abstraction of reality and not a silver bullet: no science can replace experience, domain knowledge and human intuition, as our work in risk management for the fi nance and corporate industry has shown us again and again.

Postscriptum: Extremes can also be positive, they are the best and worst of life.

Acknowlegements

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