Liquidity and Credit Risk

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ABSTRACT

We develop a structural bond valuation model to simultaneously capture liquidity and credit risk. Our model implies that renegotiation in financial distress is influenced by the illiquidity of the market for distressed debt. As default becomes more likely, the components of bond yield spreads attributable to illiquidity increase. When we consider finite maturity debt, we find decreasing and convex term structures of liquidity spreads. Using bond price data spanning 15 years, we find evidence of a positive correlation between the illiquidity and default components of yield spreads as well as support for downward-sloping term structures of liquidity spreads.

Credit risk and liquidity risk have long been perceived as two of the main justifications for the existence of yield spreads above benchmark Treasury notes or bonds (see Fisher (1959)). Since Merton (1974), a rapidly growing body of literature has focused on credit risk.[1] However, while concern about market liquidity issues has become increasingly marked since the autumn of 1998, [2] liquidity remains a relatively unexplored topic, in particular, liquidity for defaultable securities.[3]

This paper develops a structural bond pricing model with liquidity and credit risk. The purpose is to enhance our understanding of both the interaction between these two sources of risk and their relative contributions to the yield spreads on corporate bonds. Throughout the paper, we define liquidity as the ability to sell a security promptly and at a price close to its value in frictionless markets, that is, we think of an illiquid market as one in which a sizeable discount may have to be incurred to achieve immediacy.

We model credit risk in a framework that allows for debt renegotiation as in Fan and Sundaresan (2000). Following François and Morellec (2004), we also introduce

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uncertainty with respect to the timing and occurrence of liquidation conditional on entering formal bankruptcy. This permits us to investigate the impact of illiquidity in the market for distressed debt on the renegotiation that takes place when a firm is in distress.

It is often noted that the yield spreads that structural models generate are too low to be consistent with observed spreads.[4] Indeed, this may stem from inherent underestimation of default risk in these models. However, if prices of corporate bonds reflect compensation for other sources of risk such as illiquidity, then one would expect structural models to overprice bonds.[5]

Furthermore, it is also noted that the levels of credit spreads that obtain under most structural models are negligible for very short maturities, which is inconsistent with empirical evidence.[6] Again, this result holds only if the main determinant of short-term yield spreads is default risk. Yu (2002) documents the virtual impossibility of reconciling historical credit rating transition matrices to short-term yield spread data, without resorting to additional sources of risk.[7] Because our model implies nontrivial liquidity premia for short maturities, it can therefore help align structural models with this stylized fact.

We make two important assumptions about liquidity. First, when the firm is solvent, the bondholder is subjected to random liquidity shocks. Such shocks can reflect unexpected cash constraints or a need to rebalance a portfolio for risk management purposes. With a given probability the bondholder may have to sell his position immediately. The realized price is assumed to be a (stochastic) fraction of the price in a perfectly liquid market, where the fraction is modeled as a function of the random number of traders active in the market for a particular bond. We allow the probability of a liquidity shock to be a random variable that is correlated with asset value, our model's main determinant of default risk.

The supply side of the market is an endogenous function of the state of the firm and the probability of liquidity shocks. When there is no liquidity shock, the bondholder still has the option to sell if the price he can obtain is sufficiently high. A bondholder can avoid selling at a discount by holding the bond until maturity. However, he will sell preemptively if the proceeds from a sale outweigh the expected value of waiting and incurring the risk of being forced to sell at a less favorable price in the future.

We analyze the comparative statics of the model with perpetual debt and find that when the main determinants of the default probability - that is, leverage and asset risk - increase, the components of bond yield spreads that are driven by illiquidity also increase.

Our model with finite-maturity debt predicts that liquidity spreads are decreasing functions of time to maturity. This is consistent with empirical evidence on markets for government securities. Amihud and Mendelson (1991) examine the yield differentials between U.S. Treasury notes and bills that differ only in their liquidity, and find that term structures of liquidity premia do have this particular shape across short maturities. Our model implies a decreasing term structure of liquidity spreads due to the upper bound on dollar losses that can arise due to liquidity shocks before a preemptive sale takes place.

Accordingly, our model makes predictions with regard to the shape of the term structure of liquidity spreads as well as to its interaction with default risk. We study these two aspects of corporate bond yield spreads for two separate panels of U.S. corporate bond data that span a period of 15 years. Controlling for credit risk, we examine the impact of two proxies for liquidity risk, namely, a measure of liquidity risk in Treasury markets and a measure of bond age. A comparison of parameter estimates across subsamples constructed along credit ratings documents a positive correlation between default risk and the size of the illiquidity spread. Second, we find support for a downward-sloping term structure of the liquidity spread in one of our two data sets. Hence, our data lend support to two of the most salient implications of our theoretical model.

We also analyze the turbulent period surrounding Russia's default on its domestic ruble-denominated bonds. These findings are qualitatively consistent with our results for the full 15-year sample, and their economic significance is much higher.

The structure of this paper is as follows. Section I presents a model of perpetual debt and describes our framework for financial distress and illiquidity. Section II examines comparative statics for the different components of yield spreads. The case of finite maturity bonds is discussed in Section III, which also describes the model's implied term structures for liquidity premia. Section IV reports on our empirical tests of the model's predictions and Section V concludes.

I. The Model

We now describe our framework for the valuation of risky debt and the interaction between a firm's claimants in financial distress. As a starting point, we take the model of Fan and Sundaresan (2000) (FS) which provides a rich framework for the analysis of creditor-shareholder bargaining.

We use debt-equity swaps as a model for out-of-court renegotiation. In a debt-equity swap, bondholders receive new equity in lieu of their existing bonds. Such a workout is motivated by a desire to avoid formal bankruptcy and both the liquidation costs and costs associated with the illiquidity of distressed corporate debt.

In court-supervised proceedings (Chapter 11 of the U.S. Bankruptcy Code), on the other hand, the bonds are assumed to trade until distress is resolved. Resolution of distress can either entail liquidation (Chapter 7) or full recovery after successful renegotiation. We model the outcome of renegotiation in formal bankruptcy as strategic debt service,[8] whereby bondholders in renegotiation accept a reduced coupon flow in order to avoid liquidation and thereby maintain the firm in operation.

We assume that a firm is financed by equity and one issue of debt. Initially, we focus on perpetual debt with a promised annual dollar coupon of C. The risk-free interest rate r is assumed to be constant and we rule out asset sales to finance dividends or coupon payments. We also assume that agents are risk neutral so all discounting takes place at the risk-free rate. The firm's asset value is assumed to obey a geometric Brownian motion,

$$dV_t = (\mu - \beta)\, V_t dt + \sigma V_t dW_t^v, \tag{1}$$

where μ represents the drift rate of the assets, σ denotes volatility, and W_t^v is a Brownian motion. The parameter β denotes the cash flow rate, which implies that $\beta V_t dt$ is the amount of cash available at time t to pay dividends and service debt. If this value is not sufficient, shareholders may choose to contribute new capital.

When V_t reaches the lower boundary V_S, the firm defaults. In our framework, this decision is made optimally by the shareholders.[9] In the absence of a workout, the firm enters into Chapter 11. If court-supervised renegotiations fail, the firm realizes proportional liquidation costs αV_t. While absolute priority is respected in liquidation, it may be violated during bargaining in formal reorganization.

We assume that when $V_t = V_S$, shareholders and bondholders can avoid formal bankruptcy altogether by negotiating a debt-equity swap. The terms of this deal are determined as the solution to a Nash bargaining game in which the following linear sharing rule is adopted:

$$E^{w}(V_S) = \theta v(V_S), \; B^{w}(V_S) = (1-\theta) v(V_S), \tag{2}$$

where E and B denote equity and debt values, respectively, a superscript w indexes values that result from a workout, $\theta \in [0,1]$ and $v(V_t)$ is the levered firm value.[10] We assume that the two parties have respective bargaining powers of η and $(1-\eta)$, where $\eta \in [0,1]$.

According to the FS model, the outside option of bondholders forces the firm to be liquidated immediately. However, in reality, bondholders can seldom press for immediate liquidation. In Chapter 11, negotiations can go on for years under automatic stay.[11] During this period, the firm's bonds still trade and market liquidity is still a factor for creditors. To capture this feature of financial distress, we introduce uncertainty with respect to the timing and occurrence of liquidation. Following François and Morellec (2004) (FM), we do this by assuming that liquidation only takes place if the firm's asset value remains below the default threshold longer than a court-imposed observation period. Should the firm's value recover within this period, it will exit from Chapter 11.[12]

The key implications of this assumption for our model of illiquidity are that Chapter 11 takes time and that bondholders cannot avoid exposing themselves to the risk of having to sell their holdings while the firm is in distress by forcing immediate liquidation. As a result, the position of bondholders at the bargaining table will also depend on both the expected duration in Chapter 11 and the risk of having to sell distressed debt at a discount. In order to quantify the impact of liquidity risk on out-of-court debt renegotiation, we require a detailed model of the outside option. We begin by discussing the model of formal bankruptcy in the absence of illiquidity.

Let $\mathcal{T}^L$ be the liquidation date, where liquidation occurs when the firm's value remains below V_S longer than d years. When the firm is in Chapter 11, we follow FM and assume that debt is serviced strategically. This flow is denoted $s(V_t)$. If the time in default exceeds d years, the firm is liquidated, creditors recover $(1-\alpha) V_{\mathcal{T}^L}$, and shareholders' claims are worthless. Thus, the values of debt and equity conditional on entering formal bankruptcy (indexed by a superscript b) can be written as

$$\begin{aligned} B_L^b(V_S) &= E_t\left[\int_t^{\mathcal{T}^L} e^{-r(u-t)}\left(C\cdot I_{\{V_u>V_S\}}+s(V_u)\cdot I_{\{V_u\le V_S\}}\right)du\right] \\ &\quad +E_t\left[e^{-r\left(\mathcal{T}^L-t\right)}(1-\alpha)V_{\mathcal{T}^L}\right] \end{aligned} \tag{3}$$

and

$$E_L^b(V_S)=E_t\left[\int_t^{\mathcal{T}^L} e^{-r(u-t)}\left((\beta V_u-C)\cdot I_{\{V_u>V_S\}}+(\beta V_u-s(V_u))\cdot I_{\{V_u\le V_S\}}\right)du\right], \tag{4}$$

where the subscript L indicates that the debt is perfectly liquid and $I_{\{.\}}$ is an indicator function.

Now suppose that in a workout to preempt Chapter 11, bondholders are offered new securities in lieu of their existing bonds. In equilibrium, the additional value of a successful workout is $(1-\theta^*(V_S))\,v(V_S)-B_L^b(V_S)$ for bondholders, and $\theta^*(V_S)\,v(V_S)-E_L^b(V_S)$ for shareholders. The Nash solution to the bargaining game is

$$\theta^*(V_S)=\arg\max\left\{\left(\theta v(V_S)-E_L^b(V_S)\right)^{\eta}\cdot\left((1-\theta)\,v(V_S)-B_L^b(V_S)\right)^{1-\eta}\right\}. \tag{5}$$

Note that the scope for informal debt renegotiation hinges on the costs that can be avoided by not entering into formal reorganization. So far, this encompasses only the deadweight costs of liquidation in Chapter 7, reflected in the values of $B_L^b(V_S)$ and $E_L^b(V_S)$. When we introduce illiquidity, the associated costs are also part of the bargaining surplus, directly through the outside option of bondholders and indirectly through the equity value. Note that bargaining in Chapter 11 does not help mitigate the costs of illiquidity due to the continued trading of the bonds throughout the proceedings.[13] We assume that the equity issued to creditors in a workout is perfectly liquid, allowing for full avoidance of illiquidity costs.[14] We now describe our model of illiquidity and then return to a discussion of its impact on debt renegotiation.

A. Illiquidity

Figure 1 summarizes the sequence of events that occur given that the firm has not been liquidated, that is, $t < \mathcal{T}^L$.[15] First, at equally spaced time intervals (Δt years apart), the bondholder learns whether he is forced to sell his bond due to a liquidity shock.[16] Such shocks may occur as a result of unexpected cash shortages, the need to rebalance a portfolio in order to maintain a hedging or diversification strategy, or a change in capital requirements. We denote the annualized instantaneous probability of being forced to sell by λ_t and assume that

$$d\lambda_t = \kappa\left(\zeta - \lambda_t\right) dt + \sqrt{\lambda_t}\phi dW_t^{\lambda}, \tag{6}$$

where $dW_t^{\lambda} dW_t^{v} = \rho dt$. The parameter ζ can be viewed as the long-term mean of λ_t, κ the speed of mean reversion, and ϕ a volatility parameter. By allowing for a nonzero correlation coefficient between firm value and the likelihood of liquidity shocks, we can incorporate the influence of the overall state of the economy on both a firm's credit quality of and investor vulnerability. For instance, if $\rho < 0$, then during recessions firm values would tend to decrease while liquidity shocks would become more likely.[17]

FIGURE 1

Given that the bondholder is forced to sell, the discount rate that the bondholder faces is modeled as follows. The price offered by any one particular trader is assumed to be a random fraction $\widetilde{\delta}_t$ of the perfectly liquid price B_L. We assume that this fraction is uniformly distributed on $[0, 1]$. The bondholder obtains N offers and retains the best one, where N is assumed to be Poisson with parameter γ. Hence, γ measures the expected number of offers. One may also think of γ as the number of active traders in the market for a particular type of bond. While this choice of distribution and support for the individual discounts is admittedly stylized, we retain it for simplicity. The bondholder's expected best fraction of the liquid price he will be offered is[18]

$$\bar{\delta} \equiv E\left[\tilde{\delta}_t\right] = \sum_{n=0}^{\infty} e^{-\gamma} \frac{\gamma^n}{n!} \cdot \frac{n}{n+1}. \tag{7}$$

Note that as γ tends to infinity, $\bar{\delta}$ tends to one as an ever greater number of dealers compete for the same security and the price converges to the purely liquid price.

The motivation for the randomness of $\widetilde{\delta}_t$, that is, the implicit assumption that different prices for the same security can be realized at any one time, is the same as for the occurrence of liquidity shocks: Some agents trade for hedging or cash flow reasons and may, therefore, accept to buy at a higher (or sell at a lower) price than other traders.[19]

This setup is consistent with the structure of the U.S. corporate bond market, an over-the-counter market that is dominated by a limited number of dealers, as information asymmetries can readily lead to several prices being quoted in a given market at the same time.[20]

The expected value of the bond given a forced sale is

$$E_t\left[\tilde{\delta}_t B_L(V_t)\middle|\text{forced sale}\right] = B_L(V_t)\, E\left[\tilde{\delta}_t\right] = B_L(V_t)\,\bar{\delta}, \tag{8}$$

where $E_t[.]$ denotes the conditional expectation with respect to the information available at date t, after the possible realization of a liquidity shock but before the arrival of bids from bond dealers.[21] If the bondholder is not forced to sell, he still has the option to sell, should the best offer made to him be acceptable. If he decides to sell, he receives a payment of

$$\tilde{\delta}_t B_L(V_t),$$

and if he decides not to sell, the holding value is

$$e^{-r\Delta t} E_t\left[B_I\left(V_{t+\Delta t}\right)\right]. \tag{9}$$

Hence, just prior to t (i.e., at $t-$, at which point the value of the firm is known but the potential liquidity shock and the number of offers are not), the expected value of the illiquid bond if the firm is solvent is

$$\begin{aligned} & E_{t-}\left[B_I(V_t)\right] \\ = \; & E_{t-}\left[\pi_t \cdot \tilde{\delta}_t B_L(V_t) + (1-\pi_t)\max\left(\tilde{\delta}_t B_L(V_t),\, e^{-r\Delta t} E_t\left[B_I\left(V_{t+\Delta t}\right)\right]\right)\right], \end{aligned} \tag{10}$$

where $\pi_t = 1 - \exp\left\{-\int_{t-\Delta t}^{t} \lambda_s ds\right\}$ denotes the probability of a liquidity shock. We denote by δ_t^* the reservation price fraction above which the bondholder will decide to sell at time t and below which he will keep his position until the next period unless he faces a liquidity shock. This notation allows us to rewrite

$$E_{t-}\left[\max\left(\widetilde{\delta}_t B_L(V_t), e^{-r\Delta t}E_t\left[B_I(V_{t+\Delta t})\right]\right)\right],$$

as

$$\begin{aligned} & E_{t-}\left[\widetilde{\delta}_t B_L(V_t)\, I_{\widetilde{\delta}_t>\delta_t^*} + e^{-r\Delta t}E_t\left[B_I(V_{t+\Delta t})\right] I_{\widetilde{\delta}_t\le\delta_t^*}\right] \\ = \; & B_L(V_t)\, E_{t-}\left[\widetilde{\delta}_t I_{\widetilde{\delta}_t>\delta_t^*}\right] + P\left[\widetilde{\delta}_t \le \delta_t^*\right] e^{-r\Delta t}E_t\left[B_I(V_{t+\Delta t})\right]. \end{aligned} \tag{11}$$

The critical value for the offered price fraction $\tilde{\delta}_t$, above which the bondholder will decide to sell, is

$$\delta_t^* = \frac{e^{-r\Delta t}E_t\left[B_I(V_{t+\Delta t})\right]}{B_L(V_t)}. \tag{12}$$

This level equates the value of selling voluntarily with the value of waiting for another period Δt.

B. Illiquidity and Workouts

We now revisit the renegotiation process of a firm in distress when the debt of the firm trades in imperfectly liquid markets. Suppose the firm defaults at $V_t = V_S$, and subsequently a successful workout takes place. Then, the values of the firm's securities are

$$\begin{aligned} E_I^w(V_S) &= \theta_I^*(V_S)\, v(V_S) \\ B_I^w(V_S) &= (1-\theta_I^*(V_S))\, v(V_S), \end{aligned} \tag{13}$$

where subscript I indicates that the values derive from an illiquid market. The sharing rule, $\theta_I^*(V_S)$, is now the outcome of the modified bargaining problem

$$\theta_I^*(V_S) = \arg\max\left\{\left(\theta v(V_S) - E_I^w(V_S)\right)^{\eta}\cdot\left((1-\theta)\, v(V_S) - B_I^w(V_S)\right)^{1-\eta}\right\}. \tag{14}$$

Equation (14) makes it clear that the outside options of both parties depend on the impact of illiquidity on bond prices.

Unfortunately, we are unable to derive closed-form solutions for bond prices in the above setting. In order to compute security values, we rely on the Least Squares Monte

Carlo (LSM) simulation technique suggested by Longstaff and Schwartz (2001). This methodology allows us to deal with the inherent path dependence of our model of financial distress, the two correlated sources of uncertainty, and the "early exercise" feature of the bondholder's selling decision. A detailed description of the solution method is available in Appendix B.

C. Decomposing the Yield Spread

In order to quantify the influence of illiquidity on bond valuation, we focus on yield spreads; the difference in corporate bond yields and those of otherwise identical perfectly liquid risk-free securities. Consider $s_I = y_I^w - r$, the yield spread on an illiquid bond when a workout is a possible vehicle for reorganization given financial distress. Let y_L^w be the yield on a bond with the same promised cash flows in a perfectly liquid market. Note that the actual payoffs may not be identical across all states of the world since in a workout, bargaining is influenced by illiquidity. To measure the extent to which this interaction influences bond values, we also compute $y_L^{'}$, the yield on a hypothetical liquid bond with cash flows that are identical to the illiquid bond, both when the firm is solvent and when it is in distress. The spread on the illiquid bond can now be decomposed into three components:

$$\begin{aligned} s_I &= s_1 + s_2 + s_3 \\ &= \left(y_I^w - y_L^{'}\right) + \left(y_L^{'} - y_L^w\right) + \left(y_L^w - r\right). \end{aligned} \tag{15}$$

The first component, s_1, isolates the effect of liquidity shocks and the resulting trades on bond prices, in that it represents the difference in yield between two securities with the same cash flows (save illiquidity costs). However, illiquidity influences bargaining in distress. Accordingly, the second component, s_2, measures the difference in yield between two hypothetical liquid securities whose cash flows differ only by the difference between sharing rules in workouts due to the illiquidity of bonds in formal bankruptcy. Hence, s_1 can be considered a "pure" liquidity spread, and s_2 a measure of the interaction between liquidity and credit risk. Finally, s_3 measures the default risk of the firm in a perfectly liquid setting.

II. Comparative Statics

Table I summarizes the numerically estimated comparative statics. As we show in Section III below, the actual levels of yield spreads and their components for very long-term debt may differ significantly from those for realistic maturities. Hence, we first concentrate on the qualitative implications of the model before providing its extension to finite maturity debt. One key parameter is the bargaining power of shareholders, which influences how bond values respond to changes in many of the other parameters. Rather than treating this parameter in isolation, we consider two sets of comparative statics, one for situations characterized by high shareholder bargaining power ($\eta = 0.75$, Panel A) and one for high bondholder bargaining power given distress ($\eta = 0.25$, Panel B).

TABLE I

The long-run mean of the instantaneous liquidity shock probability, ζ, is distinctly positively correlated with the nondefault components of the spreads. Both the pure illiquidity spread, s_1, and the workout spread, s_2, increase, regardless of the relative bargaining powers of bondholders and shareholders. Since the default component of the yield spreads remains unaffected, the total spread increases in ζ.

The impact of the mean number of dealers, γ, is also clear: It decreases both s_1 and s_2. Interestingly, both ζ and γ influence the default policy of the firm. The higher the liquidity shock probabilities and the lower the number of active dealers, the earlier the shareholders will want to default. This will tend to decrease the liquidity spread and increase the workout spread. However, this effect is not strong enough to fully counter the direct effect on the illiquidity spread by the increased likelihood of a shock (higher ζ) or by a bigger discount conditional on selling (lower γ).

The effect of leverage, as measured by the annual coupon amount C, is more subtle. The higher the leverage, the higher the default threshold. This tends to increase the default spread s_3. A higher default probability implies that a workout with an ensuing debt-equity swap becomes more likely. As the expected lifetime of the bond decreases, it decreases the liquidity spread component s_1, due to a reduction in the risk of being exposed to liquidity shocks while solvent. However, in the absence of a workout, s_1 would not decrease. If shareholders have bargaining power in a workout, they can extract concessions from bondholders that are equivalent to a fraction of the illiquidity costs that would be incurred in Chapter 11. Thus, the spread component s_2 increases.

Overall, the effect of an increase in the workout spread dominates and s_1+s_2 increases in leverage under both bargaining power scenarios.

The effect of asset risk is similar to that of leverage save for one major difference. Although an increase in asset risk makes a workout more likely, thus increasing s_2, it also increases the optionality of equity. With a higher level of risk, shareholders may be willing to keep the firm alive longer to benefit from the possible future upside. As a result, the default threshold is lower for a given level of leverage.[22] Thus, the liquidity spread $s_{1,}$ which decreases with leverage, actually increases. In aggregate, therefore, s_1+s_2 increases in asset risk both for high and low shareholder bargaining power.

With respect to cash flow rate β, the higher the β the lower the growth rate of the firm and the higher the risk of distress. This tends to have a negative effect on the liquidity component s_1. However, any increase in β also decreases V_S (shareholders receive more dividends and are willing to keep the firm afloat longer), which in turn offsets the increase in distress probability. In short, the effect of β on s_1 is positive, and on s_2 is negative, and the overall effect is an increase in s_1+s_2.

An increase in liquidation costs, α, increases the default threshold and thus the probability of entering into a workout.[23] When the bargaining power of shareholders is high, there is a stronger incentive for shareholders to default earlier. In this scenario, an increase in α yields a faster decrease in s_1. The workout component, s_2 , also increases when the bargaining power is high, but remains unaffected in the opposite scenario. The overall effect turns out to be a net increase in s_1+s_2, except for an increase in α from high levels when η is high.

The exclusivity period in Chapter 11, d, increases the illiquidity costs to be shared in out-of-court bargaining and thus the spread component s_2. In turn, it gives shareholders an incentive to default earlier, and hence s_1 decreases in d. The net effect is an increase in (s_1+s_2), particularly when η is high. Thus, d increases yield spreads through both the nondefault and default components.

Surprisingly, the effect of the correlation between the asset value and the probability of a liquidity shock on the spread components proves to be relatively weak. When the bargaining power of shareholders is elevated, the workout spread is inversely related to ρ. Intuitively, when a workout occurs in times of frequent liquidity shocks, the impact of illiquidity on the workout is greater. When shareholders have low levels of bargaining power, the effect is the same but less pronounced. The relationship between ρ and s_1, the pure liquidity spread, is ambiguous, and weaker still. Note that the comparative statics for the other parameters rely on neither the size nor the sign of the correlation coefficient.

In summary, variables that are positively related to the default component of the spread also tend to increase the sum of the pure liquidity spread, s_1, and the workout spread, s_2. The only exception is an accrual in already high liquidation costs when shareholders enjoy high levels of bargaining power. While the liquidity component may decrease at the onset of distress, the increase in spread due to the influence of the illiquidity of distressed debt on bargaining in a workout does tend to more than compensate for it.

III. Term Structures of Liquidity Premia

The assumption of infinite debt maturity is obviously restrictive if we wish to gauge the quantitative output of our model. To allow us to relax this assumption without making the problem intractable, we rely on a debt structure proposed by Leland and Toft (1996). We assume that the firm continuously issues new bonds with principal p, coupon c, and maturity T, at which point the principal is also repaid. The rate of issuance of new debt is $p = \frac{P}{T}$, where P is the total par value of debt outstanding. The main value of this assumption for our analysis is that the firm has bonds outstanding whose maturities range from 0 to T, and this allows us to determine the full-term structure of bond yield spreads. In addition, this assumption implies that the total debt service $\left(C + \frac{P}{T}\right)$ of the firm is time independent, which, in turn, implies that the endogenous default threshold does not depend on time either.[24]

As before, we solve the valuation problem by Least Squares Monte Carlo, as described in Appendix B. Appendix C reviews the necessary results from François and Morellec (2004) and Leland and Toft (1996).

Figures 2 to 5 below provide a visual summary of the results. As a benchmark, in Figure 2 we begin by plotting our model's liquidity spreads as a function of time to maturity in the absence of default risk. Consistent with the results of Amihud and Mendelson (1991), a decreasing and convex shape obtains for the term structure of liquidity spreads.[25]

FIGURE 2

Figure 3 plots the illiquidity spread as a function of the maturity of the bond for different levels of γ, the mean number of active dealers. This graph clearly shows how taking the maturity of the bond into account is crucial for computing a liquidity spread. Moreover, we see that the spreads can be substantial, especially for short-term bonds. Indeed, the decreasing and convex shape of the term structure of liquidity spreads that emerges in this figure can help reconcile structural models with the nontrivial short-term spreads we observe in the marketplace.

FIGURES 3,4

Figure 4 plots term structures of liquidity spreads for various levels of the annualized intensity of a liquidity shock, ζ. Again, we find that short-term spreads can be substantial and that the term structure is downward sloping.

FIGURE 5

Figure 5 plots the proportions of the total yield spread that are attributable to default risk and liquidity risk. In particular, the figure emphasizes the importance of illiquidity on short-term spreads: For bonds with less than two years to maturity, illiquidity comprises the main component of the spread. For long-term bonds, the illiquidity component stabilizes (for this particular set of parameters) at about 8% of the total spread.

IV. Empirical Analysis

In this section we ask how corporate bond data compare to our model's predictions. First, we investigate whether liquidity spreads and credit spreads are related in the data. Second, we wish to test whether the slope of the term structure of liquidity spreads is negative. While full structural estimation of our model lies beyond the scope of this paper, we test the model's implications by regressing bond yield spreads on two sets of variables; one that controls for credit risk, and one that proxies for liquidity risk. We then compare parameter estimates across subsamples defined along credit ratings and bond maturities.

We estimate the following panel regression with fixed effects for the bond spread y_{it} of issue i at time t:

$$\begin{aligned} y_{it} &= \alpha_i + \beta_1 VIX_t + \beta_2 SPRET_t + \beta_4 SLOPE_t + \beta_5 r_{it} + \beta_6 DEFPREM_t \\ &\quad + \beta_7 OTR_{it} + \beta_8 TLIQ_t + \varepsilon_{it}, \\ \text{where } \varepsilon_{it} &= \rho\varepsilon_{i,t-1} + \eta_{it}. \end{aligned} \tag{16}$$

We assume that the disturbances η_{it} are independently identically distributed.[26]

In equation (16), VIX is a proxy for overall equity market volatility, $SPRET$ is the market return, $SLOPE$ is the difference between long and short government yields, r_{it} is the risk-free rate with the same maturity as the corporate bond, OTR is a dummy for younger bonds, and $TLIQ$ is a proxy for Treasury market liquidity. Note that this specification allows for autocorrelation in the panel data for which we find strong evidence.[27] We run this regression on two panels that we construct from separate data sources, namely, monthly observations from Datastream and NAIC transactions data. The first panel consists of 522 zero-coupon bond issues that yield a total of 35,198 monthly price observations. The data span the period 1986 to 1996. The NAIC data complete the first panel by covering the period 1996 to 2001 with 37,861 transaction prices for bonds traded by U.S. insurance companies. Table II furnishes descriptive statistics for the two samples.[28] Note that the second sample demonstrates a much more even coverage of credit rating categories than the first, which is concentrated in very high quality issues. As a result, the level and variation of yield spreads are higher in the NAIC data. A priori, we expect the NAIC database to offer a more promising study of the relationship between credit quality and the illiquidity components of bond spreads.

TABLE II

We calculate spreads as the difference between the risky bond yield and the risk-free rate obtained by the Nelson and Siegel (1987) procedure. Appendix D contains a more detailed description of the construction of spreads.

TABLE III

Table III provides an overview of the expected relationships between our liquidity and nonliquidity proxies and bond yield spreads. Again, we utilize five variables in order to capture variations in the bond yield spreads that are not attributable to liquidity risk. Specifically, we include measures of stock market return and volatility, two Treasury term structure variables, and a metric for the aggregate default risk in the economy. We then add a proxy for the liquidity of each individual issue, and a proxy for the liquidity of the fixed income markets as a whole.

A. Results

The issues we wish to examine are whether there is a relationship between the illiquidity and credit risk components of spreads, and whether the term structure of liquidity spreads is decreasing. We address these questions by comparing parameter estimates for our liquidity proxies in subsamples defined by credit ratings and maturities. Bonds with a maturity that exceeds the average maturity of 12 years are placed in a subsample of "long" bonds. We present the regression results in Table IV for the zero-coupon bonds and in Table V for the NAIC data.

TABLE IV, V

For the zero-coupon data set, the high rating category contains AAA bonds and the low category contains the remaining bonds.[29] For the regression that consists of all yield spread observations, we note that almost all the nonliquidity-related coefficient estimates are signed consistent with our expectations and with the implications of structural credit risk models.[30] Stock market volatility is significantly and positively associated with the level of yield spreads, except for AAA bonds, whose spreads are unlikely to be driven mainly by default risk. Structural models of default risk derive high equity volatilities from high leverage.

We obtain a significant negative relationship between S&P 500 returns and yield spreads. A positive return is likely to be associated with a decrease in leverage and consequently, in the default probability and spread. This finding is robust across all subsamples except for long maturity bonds, for which the parameter estimates are still negative but insignificant.

The level of the risk-free interest rate is always negatively related to the spread levels, in line with Duffee (1999). The relationship is more marked for firms with a low credit rating and for bonds with a short maturity. The $SLOPE$ variable behaves similarly.

Not surprisingly, the aggregate market default premium, as measured by the spread between Moody's Baa and Aaa yield indices, is positively related to the level of individual bond spreads. Again, the impact is larger for issues with a lower credit rating.

The signs of the OTR dummy and $TLIQ$ are consistent with our interpretation that they proxy for liquidity. On average, a recently issued bond in the full sample can expect to trade at around 10 basis points less than if it were more seasoned. A greater illiquidity premium in Treasury markets translates to higher yield spreads in the corporate bond market. However, this effect is weaker since a 10 basis point-increase in $TLIQ$ tends to increase yield spreads by little more than one basis point. The OTR parameter estimates are significant in all regressions, and the $TLIQ$ estimates are significant in all cases but one.

The parameter estimate for the OTR dummy is more than three times larger in the subsample of bonds with a low credit rating relative to the subsample of AAA bonds. This suggests that off-the-run credit-risky bonds have to reward their holders with an additional yield, which can be in excess of three times higher than the corresponding extra yield for high credit quality bonds. Similarly, the impact of $TLIQ$ is larger in the low rating sample by a similar magnitude. Both of these findings support our model's finding of a positive relationship between credit and liquidity risk.

We now turn to a discussion of the results for the NAIC transaction data. We run the same panel regression for the full sample and for the subsamples, again defined by credit rating and maturity. In the full sample, the results for the default risk proxies are similiar to those for the zero-coupon bond data, with the exception of the market return. Surprisingly, the coefficient estimate for the S&P 500 return is positive and significant.[31] The other variables enter with the expected signs.

The OTR dummy enters with a negative sign and is statistically significant. The coefficient estimate is greater than in the zero-coupon bond sample; on average, a newly issued bond trades at almost 30 basis points less than an older one, after controlling for default risk. However, the average credit quality in this sample is much lower. When we consider the more comparable subsample of bonds with S&P ratings between AAA and $AA-$, we find that the coefficient estimate is close to 10 basis points, which, in turn, is remarkably close to the estimate for the zero-coupon bonds. In the next rating category ($A+$ to $BBB-$), the coefficient estimate roughly trebles. For $BB+$ to $B-$ ratings, the estimate increases further, to the extent that younger bonds have spreads that are lower, on average, by over 80 basis points. In the $CCC+$ to D category, the coefficient is positive but insignificant.

The results for the $TLIQ$ variable are somewhat more difficult to interpret. For the standard regression in the full sample, the coefficient is negative and marginally significant. When we look across the subsamples, the coefficient estimate is negative for the two highest rating categories. As credit quality declines, the coefficient estimate becomes positive and is largest for the poorest quality bonds. One explanation for this variable's surprising negativity for high quality bonds may be that it is correlated with $DEFPREM$, the market default premium (the sample correlation coefficient is 0.54 between 1996 and 2001, while it is only 0.17 between 1986 and 1996). If we drop $DEFPREM$ in the regression, the $TLIQ$ variable behaves as for the zero-coupon bond data. For the entire sample, $TLIQ$ is positive, and significant and, with the exception of one rating category, it is uniformly increasing in the default risk of the bonds.

In addition, we perform a case study of the turbulent market conditions prevalent in the late summer and autumn of 1998 that surrounded Russia's default on its bonds. We consider the first of the above regression models for corporate bond spreads during three periods. Specifically, we study first the period from January 1, 1998 to August 14, 1998 - the Friday preceding the Monday on which the Russian government defaulted on its debt. Second, we examine the crisis period, which we define as August 17, 1988 to November 20, 1988. We then consider a post-crisis period from the November 23, 1998 to October 29 of the following year. Table VI reports the results for the regressions for each of the three periods for different rating categories. Note that the much smaller sample sizes here cause us to lose power. As a result, we do not obtain statistical significance for the illiquidity proxy (OTR). However, it is still interesting to consider the behavior of the coefficient estimate, which is consistently negative as in the full sample regressions for the zero-coupon and NAIC data sets. For all bonds, it roughly trebles during the crisis period and then drops to a level about 50% higher than the pre-crisis level. For investment grade bonds, the pattern is the same but with a less dramatic increase during the crisis period than for the speculative grade subsample (the coefficient estimate jumps to -100 and -158 basis points, respectively).

TABLE VI

Overall, the results for the two data sets and for the Russian default case study suggest a clear, positive correlation between the default and liquidity components of bond yield spreads. This is consistent with our model when shareholders have bargaining power in a workout. It is interesting to note that the link between the two spread components is apparent in both data sets, notwithstanding their differences in coverage of credit quality and time.

Turning to the shape of the term structure of liquidity spreads, we find in the first data set (see Table 4) that the impact of the OTR dummy differs across bond maturities. Short-term bonds benefit three times more from being on the run than long bonds, suggesting that the liquidity component of yield spreads diminishes with maturity.[32]

The difference between the parameter estimates for $TLIQ$ is not statistically significant. A similar analysis on the NAIC transactions data (not reported here) reveals no discernible pattern across maturity subgroups.

V. Concluding Remarks

We develop a model to illustrate the impact of liquidity risk on the yield spreads of corporate bonds. The model has a number of interesting features. Our main qualitative findings are that levels of liquidity spreads are likely to be positively correlated with credit risk and that they should be decreasing functions of time to maturity.

Another result is that, for reasonable parameter inputs, the model is able to generate substantial yield spreads, even for short maturities. This addresses a common criticism of structural bond pricing models and helps reconcile them with empirical evidence.

In our empirical analysis, we find that U.S. corporate bond data support our model's prediction that liquidity spreads are positively correlated with the likelihood of default. We also find support, albeit weaker, for liquidity spreads decreasing with time to maturity.

The model developed in this paper is a partial equilibrium one since it takes the holdings of the bondholder as given. We do not model the bondholder's initial bond purchase decision. In addition, bondholders may demand contractual features in bonds to mitigate illiquidity costs, for example, embedded put options that allow the holder to sell the bond back to the issuer and thereby provide partial insurance against the consequences of a liquidity shock.[33] In our framework, the put allows bondholders to avoid costs associated with liquidity shocks while the firm remains solvent. However, the put becomes less valuable as the firm approaches distress and the illiquidity component of yield spreads becomes sizeable again. An empirical study of putable debt would prove interesting in this respect.

Notes

[1]See for example Black and Cox (1976), Kim, Ramaswamy, and Sundaresan (1993), Shimko, Tejima, and van Deventer (1993), Nielsen, Saá-Requejo, and Santa-Clara (1993), Longstaff and Schwartz (1995), Anderson and Sundaresan (1996), Jarrow and Turnbull (1995), Lando (1998), Duffie and Singleton (1999), and Collin-Dufresne and Goldstein (2001).

[2]Indeed, the BIS Committee on the Global Financial System underlines the need to understand the sudden deterioration in liquidity during the 1997 to 1998 global market turmoil. See BIS (1999).

[3]Some recent empirical work with reduced form credit risk models allows for liquidity risk. Examples include Duffie, Pedersen and Singleton (2003), Janosi, Jarrow and Yildirim (2002) and Liu, Longstaff and Mandell (2006).

[4]See, for example, Jones, Mason, and Rosenfeld (1984) and Huang and Huang (2002).

[5]This view has been pursued in recent work by Huang and Huang (2002), who measure the amount of credit risk compensation in observed yield spreads. Specifically, they calibrate several structural risky bond pricing models to historical data on default rates and loss given default. They find that for high grade debt, only a small fraction of the total spread can be explained by credit risk. For lower quality debt a larger part of the spread can be attributed to default risk.

[6]This argument is one of the motivations for the article by Duffie and Lando (2000).

[7]His study is based on the reduced-form model of Jarrow, Lando, and Yu (2005), in which default occurs at the first jump in a Cox process. Thus, the lack of jumps to default in the typical structural model cannot alone explain the underestimation of yield spreads at short maturities.

[8]See Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000), and François and Morellec (2004) for a more detailed discussion of this vehicle for modeling renegotiation.

[9]The ex post optimal default threshold needs to be determined numerically in our setting.

[10]The levered firm value equals the asset value less expected liquidation costs. For simplicity, we do not consider corporate taxes.

[11]Automatic stay describes an injunction issued automatically upon the filing of a petition under any chapter of the Bankruptcy Code by or against the debtor. This injunction prohibits collection actions against the debtor, providing him relief so that a reorganization plan can be structured without disruption.

[12]The main impact of this assumption on security values in François and Morellec (2004) is that the value of the firm over which claimants bargain depends on the length of time that the firm is expected to spend in Chapter 11 and the probabilities of liquidation and recovery, respectively.

[13]Hence, the agreed reduction in debt service flow under Chapter 11 will not be affected by the continuing illiquidity during the proceedings.

[14]Note that this particular choice of reorganization vehicle is not crucial. The key assumption is that bondholders receive new and less illiquid securities than their current holdings. Thus, we could accommodate exchange offers in which bondholders receive a mix of new bonds and an equity component.

[15]The Longstaff (1995) model lies close in spirit to ours. He measures the value of liquidity for a security as the value of the option to sell it at the most favorable price over a given window of time. Although our results are not directly comparable because he derives upper bounds for liquidity discounts for a given sales-restriction period, his definition of liquidity approximates our own.

To date, Tychon and Vannetelbosch (2005) is, to our knowledge, the only paper that models the liquidity of corporate bonds endogenously. They use a strategic bargaining setup in which transactions take place because investors have different views about bankruptcy costs. Although some of their predictions are similar to ours, their definition of liquidity risk differs significantly. Notably, as their liquidity premia are linked to the heterogeneity of investors' perceptions about the costliness of financial distress, their model predicts that liquidity spreads in Treasury debt markets should be zero.

[16]Note that we do not model the bondholder's equilibrium holdings of cash vs. bonds. We model a single bondholder with unit holdings of the bond.

[17]Fund managers are often subject to constraints on the credit rating of bonds they hold in their portfolio. Thus, as the credit quality of a bond declines, the manager will become more likely to sell it, consistent with a negative ρ.

[18]Details of the calculations can be found in Appendix A.

[19]We assume here that the demand side of the market is unaffected by events that impact bond value. However, it is possible to extend our framework to allow for offer distributions that are dependent on the risk return characteristics of a bond. Risk-averse bond dealers would demand steeper discounts as the credit quality of the bond declines. Results for such a specification are qualitatively similar to those we obtain in this much simpler setting.

[20]See for example Schultz (1998) and Chakravarty and Sarkar (1999).

[21]The distribution of offers is assumed constant over time so that $E_t\left[\tilde{\delta}_t\right] = E\left[\tilde{\delta}_t\right] = \bar{\delta}$. An alternative way to introduce a correlation between asset values and market liquidity would be to adopt a specification for γ similar to the one we choose for λ_t in (6).

[22]See also, for example, Leland (1994), Fan and Sundaresan (2000).

[23]See also Fan and Sundaresan (2000) and François and Morellec (2004).

[24]Given that we need to compute the ex post optimal default policy numerically, solving the problem of the bondholder (taking into account the path dependent nature of our model of distress together with bargaining and the correlated dynamics of two state variables) would be virtually impossible if the default policy were a general function of time.

[25]Note, however, that their study only considers the short end of the term structure.

[26]See Baltagi and Wu (1999) for a detailed description of this panel model.

[27]A test developed by Wooldridge (2001) was used.

[28]We have excluded bonds with less than one year to maturity because of the extreme

sensitivities of short bond spreads to small changes in price and thus to noise in the data. We also exclude all bonds with option features and sinking funds.

[29]The ratings in the second category range from AA+ to BBB+.

[30]Note that the reported R-squares measure the explained variation in yield spreads not captured by the fixed effects. Take for example the first regression in Table IV. The reported R-square is 6.39%. If this regression had been run instead as a standard pooled OLS regression with issuer dummies, the R-square would be more aligned with those reported in previous studies such as Campbell and Taksler (2003), in the range of 30% to 40%.

[31]This result is consistent with the findings of Campbell and Taksler (2003). They find that although equities performed strongly during the late 1990s, yields on corporate bonds relative to Treasuries increased. They attribute this difference in performance to an increase in idiosyncratic volatility.

[32]The cut-off for long and short maturity bonds was taken to be approximately the median maturity. To see whether this choice is critical to our results, we rerun regressions for maturity segments ranging from 0-2, 2-4 and so on up to 28-30 years. The coefficient estimates for the OTR variable indicate that newly issued bonds with less than 2 years to maturity on average have yield spreads lower by about 60 basis points than their seasoned counterparts. This yield differential decreases smoothly for the next three maturity segments to reach approximately 10 basis points. For bonds longer than 8 years, the yield differential oscillates between 5 to 15 basis points. No clear pattern for the TLIQ coefficients emerges.

[33]We thank the referee for pointing this out.

Appendix A : Expected Discounts

The expected best fraction of the liquid price that the seller will be offered is

$$E\left[\tilde{\delta}_t\right] = \sum_{n=0}^{\infty} P\left(N = n\right) \int_0^1 \delta f^n\left(\delta\right) d\delta,$$

where the density $f^n(x)$ is the probability that x is the best price fraction obtained, given n offers. Given only one offer, for a uniform distribution the probability of obtaining a fraction of less than x is

$$F\left(x\right) = x,$$

where F is the cumulative distribution. Thus, with n independent offers the probability of obtaining no offer higher than x is

$$\left(F\left(x\right)\right)^n = x^n$$

and the desired density function f^n is

$$f^n\left(x\right) = \frac{\partial\left(F\left(x\right)\right)^n}{\partial x} = nx^{n-1}.$$

Given that the number of offers is Poisson with parameter γ, we have

$$\begin{aligned} E\left[\tilde{\delta}_t\right] &= \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!} \cdot \int_0^1 n\delta^n d\delta, \\ &= \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!} \cdot \frac{n}{n+1}. \end{aligned}$$

In order to compute the value of an illiquid bond, we need to solve

$$E_{t-}\left[\tilde{\delta}_t \cdot I_{\tilde{\delta}_t > \delta_t^*}\right]$$

and

$$P\left(\tilde{\delta}_t > \delta_t^*\right) = E\left[I_{\tilde{\delta}_t > \delta_t^*}\right].$$

Recall

$$P(N=n)=e^{-\gamma}\frac{\gamma^n}{n!},$$

and conditional on n offers our assumption of uniformly distributed offers yields the following density for the price fraction δ offered:

$$f^n(\delta)=n\delta^{n-1}.$$

It therefore follows that

$$\begin{aligned}
E\left[I_{\tilde{\delta}_t>\delta_t^*}\right] &= \sum_{n=0}^{\infty}P(N=n)\,E\left[I_{\tilde{\delta}_t>\delta_t^*}\,|N=n\right] \\
&= \sum_{n=0}^{\infty}P(N=n)\int_{\delta_t^*}^{1}n\delta^{n-1}d\delta \\
&= \sum_{n=0}^{\infty}e^{-\gamma}\frac{\gamma^n}{n!}\left(1-(\delta_t^*)^n\right),
\end{aligned}$$

and

$$\begin{aligned}
E_{t-}\left[\tilde{\delta}_t\cdot I_{\tilde{\delta}_t>\delta_t^*}\right] &= \sum_{n=0}^{\infty}P(N=n)\,E\left[\tilde{\delta}_t\cdot I_{\tilde{\delta}_t>\delta_t^*}\,|N=n\right] \\
&= \sum_{n=0}^{\infty}P(N=n)\int_{\delta_t^*}^{1}n\delta^{n}d\delta \\
&= \sum_{n=0}^{\infty}e^{-\gamma}\frac{\gamma^n}{n!}\frac{n}{n+1}\left(1-(\delta_t^*)^{n+1}\right).
\end{aligned}$$

Appendix B : Numerical Methods for Computing Bond Prices

This appendix summarizes the numerical procedure we employ to price corporate debt and compute spreads. Our method is based on Longstaff and Schwartz (2001), in which contingent claims are priced by a combination of Monte Carlo simulation and linear regression.

The LSM Algorithm

The Least Squares Monte Carlo (LSM) technique was originally developed to price American options with finite maturity. The basic idea is the following. First, one simulates paths for the value of the underlying asset (stock price) for all time steps until maturity. The steps are chosen to be sufficiently small to minimize any discretization bias. At each intermediate time step, the continuation value for the option is estimated by regressing the discounted value of future payoffs on the current stock price. The explanatory variables in the regression include the value of the stock as well as powers or other transformations of this value (see Longstaff and Schwartz (2001) for more details). Thus, the algorithm allows one to compare the immediate exercise value of the option with its continuation value and thereby determine the optimal exercise time.

Pricing Infinite Maturity Debt

Numerical techniques such as trees, finite differences, or the above LSM technique are suitable to price finite maturity assets. In order to adapt the above setup to the pricing of infinite maturity debt, we assume that illiquidity only applies until year Θ, with Θ arbitrarily large but finite. (In practice, we choose Θ between 20 and 40 years and determine it numerically such that results using Θ or $\Theta + 1$ are almost indistinguishable).

Both time until Θ is split into N identical time steps of length Δt and n paths of the firm value V are simulated: $V_{i,t}$ for $i = 1, \dots, n$ and $t = 1, \dots, N$. At Θ, on a given path i, the price of an illiquid and of a liquid bond are identical, and are denoted $B_L(V_{i,\Theta}) = B_I(V_{i,\Theta}) = FM(V_{i,\Theta}, V_B)$, where $FM(.,.)$ is the formula for the bond price in François and Morellec (2004), and; V_B is provided in closed form in that paper.

At the previous time $(\Theta - \Delta t)$, the "continuation value" for the illiquid bond is estimated as

$$Y_i^I(V_{i,\Theta-\Delta t}) = e^{-r\Delta t} E_{\Theta-\Delta t}[B_I(V_{i,\Theta})],$$

and the expectation is computed by regressing $B_I(V_{i,\Theta}), i = 1 \ldots, n$, on $(V_{i,\Theta-\Delta t})$, $(V_{i,\Theta-\Delta t})^2$, and $(V_{i,\Theta-\Delta t})^3$.

This provides a value for equation (9) that can be inserted into (10) to determine the price of the illiquid bond at time $\Theta - \Delta t$.

These steps are repeated recursively until the initial date. If the probability of the liquidity shock $\lambda_{i,t}$ or the mean number of traders $\gamma_{i,t}$ are stochastic, additional terms are included in the regression, namely, $(x_{i,t}), (x_{i,t})^2, (x_{i,t})^3, (x_{i,t}V_{i,t}), (x_{i,t})^2 V_{i,t}, x_{i,t}(V_{i,t})^2$, and $(x_{i,t})^2(V_{i,t})^2$, where $x_{i,t} = \lambda_{i,t}$ or $\gamma_{i,t}$. These terms reflect the fact that the optimal time to sell the security depends not only on the value of the firm but also on the distribution of offers and the probability of forced sale.

Finite Maturity Debt

To obtain closed-form solutions for debt prices, Fan and Sundaresan (2000) and François and Morellec (2004) remove time dependence by assuming that debt is perpetual. The authors are able to obtain closed-form solutions for debt prices. An alternative approach to avoiding time dependence while preserving finite maturities is to assume that the firm's capital structure is stationary (Leland and Toft (1996)). This is achieved by assuming that the firm continuously issues debt such that maturing bonds are replaced with new issues. We rely on this assumption in our section on finite maturity debt.

In order to be consistent with the section on infinite maturity debt, we maintain the default-triggering mechanism of François and Morellec (2004) and extend the model of Leland and Toft (1996) to allow for Chapter 11-type defaults (see Appendix C). As in the case of infinite maturity debt, we assume that after a certain time Θ, illiquidity ceases to affect security prices. We choose Θ to be much larger than the maturity T of the debt we want to price, in order to avoid numerical distortions. At Θ, the price of all debt outstanding and the liquidation barrier are determined as in Leland and Toft (1996), with the extension that liquidation is triggered by the length of *time spent below* the barrier and not by the *first passage time to* the default barrier. For any intermediate time period between $t = 0$ and $t = \Theta$, prices are computed by LSM using the same explanatory variables in the regressions as for infinite maturity debt.

Appendix C: Finite Maturity Debt in the FM Model

In order to compute the default policy in the case of finite maturity debt, we need to extend the FM model. Assuming no taxes and perpetual debt, their equity value can be written as

$$E(d,c) = V - V_S \left(\frac{V}{V_S}\right)^{-\xi} - \frac{c}{r}\left(1 - \left(\frac{V}{V_S}\right)^{-\xi}\right) + \eta R(d) \left(\frac{V}{V_S}\right)^{-\xi},$$

where $R(d)$ is the renegotiation surplus at the outset of financial distress (see equation (12) in FM).

Leland and Toft (1996) allow for finite debt maturity while retaining a stationary debt structure. They achieve this by assuming that a firm sells a constant amount of new straight-coupon debt with T years to maturity. When a bond matures, it is replaced by a new issue with identical contractual terms. It follows from these assumptions that the value of the debt service on current debt is

$$\frac{c}{r} + \left(P - \frac{c}{r}\right)\left(\frac{1 - e^{-rT}}{rT} - I(T)\right),$$

where

$$I(T) = \frac{1}{T}\int_0^T e^{-rs} F(s)\, ds$$

and where F and G are given by equations (4) and (5) in Leland and Toft (1996). We can therefore write the value of equity in our framework as

$$\begin{aligned} E(d,c) &= V - V_S \left(\frac{V}{V_S}\right)^{-\xi} - \frac{c}{r}\left(1 - \left(\frac{V}{V_S}\right)^{-\xi}\right) && (17) \\ & && (18) \\ &\quad - \left(P - \frac{c}{r}\right)\left(\frac{1 - e^{-rT}}{rT} - I(T)\right) + \eta R(d)\left(\frac{V}{V_S}\right)^{-\xi}. && (19) \end{aligned}$$

It is this formula that allows us to numerically compute the default threshold V_S.

Appendix D : Spread Construction

Spreads are defined as the difference between the yield on a corporate bond and the yield on a U.S. Treasury bond with the same maturity. Given that there do not exist bonds for all maturities, we choose to construct a whole term structure of risk-free rates from existing bond prices for each month-end from January 1986 to December 1996 (132 months). For the NAIC data, we perform the same procedure on the relevant trading days.

We use the Nelson and Siegel (1987) algorithm to obtain a smooth yield curve from zero-coupon bonds. This procedure is a four-parameter yield curve calibration method whose flexible specification allows us to replicate most term structure shapes usually observed on the market. Formally, the yield at time t on a bond with maturity T is given by

$$R(t,T) = \beta_0 + (\beta_1 + \beta_2)\frac{1 - \exp(-T/\beta_3)}{T/\beta_3} - \beta_2 \exp(-T/\beta_3).$$

Using risk-free, zero-coupon bonds (mainly strips) to derive the benchmark curves enables us to obtain a nearly perfect fit of observed riskless rates by maximum likelihood. However, we find that the Nelson-Siegel procedure is overparametrized for zero-coupon bonds and generates wide differences in the parameter estimates, in spite of only mild variations in their initial values. We therefore impose a restriction on the first parameter, the only one with a clear economic interpretation. More precisely, the first parameter represents the yield of a perpetual risk-free bond $R(t,\infty)$. We approximate it by the 30-year U.S. Treasury rate to obtain a consistent and robust set of optimal parameters. The constraint yields positive forward rates for all maturities and throughout all observation periods, thereby sidestepping one of the main criticisms of the algorithm. For each trading day, we exclude risky bonds, whose maturity falls outside the range spanned by the risk-free bonds, to avoid the imprecisions of the interpolation procedure outside this range.

REFERENCES

Amihud, Yakov, and Haim Mendelson, 1991, Liquidity, maturity, and the yields on U.S. Treasury securities, *Journal of Finance* 46, 1411-1425.

Anderson, Ronald W., and Suresh Sundaresan, 1996, Design and valuation of debt contracts, *Review of Financial Studies* 9, 37-68.

Baltagi, Badi H., and Ping X. Wu, 1999, Unequally spaced panel data regressions with AR(1) disturbances, *Econometric Theory* 15, 814-823.

BIS Committee on the Global Financial System, 1999, A review of financial market events in autumn 1998.

Black, Fisher, and John C. Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351-367.

Campbell, John T., and Glen B. Taksler, 2003, Equity volatility and corporate bond yields, *Journal of Finance* 58, 2321-2349.

Chakravarty, Sugato, and Asani Sarkar, 1999, Liquidity in U.S. fixed-income markets: A comparison of the bid-ask spread in corporate, government and municipal bond markets, Federal Reserve Bank of NY staff report 73.

Collin-Dufresne, Pierre, and Robert Goldstein, 2001, Do credit spreads reflect stationary leverage ratios?, *Journal of Finance* 56, 1929-1957.

Duffee, Gregory R., 1999, Estimating the price of default risk, *Review of Financial Studies* 12, 197226.

Duffie, Darrell, and David Lando, 2000, Term structures of credit spreads with incomplete accounting information, *Econometrica* 69, 633-664.

Duffie, Darrell, Lasse Heje Pedersen, and Kenneth J. Singleton, 2003, Modeling sovereign yield spreads: A case study of Russian debt, *Journal of Finance* 58, 119-160.

Duffie, Darrell, and Kenneth J. Singleton, 1999, Modeling term structures of defaultable bonds, *Review of Financial Studies* 12, 687-720.

Fan, Hua, and Suresh Sundaresan, 2000, Debt valuation, renegotiation and optimal dividend policy, *Review of Financial Studies* 13, 1057-1099.

Fisher, Lawrence, 1959, Determinants of the risk premiums on corporate bonds, *Journal of Political Economy* 67, 217-237.

François, Pascal, and Erwan Morellec, 2004, Capital structure and asset prices: Some effects of bankruptcy procedures, *Journal of Business* 77, 387-412.

Huang, Jing-Zhi, and Ming Huang, 2002, How much of the corporate-Treasury yield spread is due to credit risk? A new calibration approach, Working paper Pennsylvania State University.

Janosi, Tibor, Robert Jarrow, and Yildiray Yildirim, 2002, Estimating expected losses and liquidity discounts implicit in debt prices, *Journal of Risk* 5, 1-38.

Jarrow, Robert, David Lando, and Fan Yu, 2005, Default risk and diversification: Theory and empirical implications, *Mathematical Finance* 15, 1-26.

Jarrow, Robert A., and Stuart M. Turnbull, 1995, Pricing derivatives on financial securities subject to credit risk, *Journal of Finance* 50, 53-85.

Jones, E. Philip, Scott P. Mason, and Eric Rosenfeld, 1984, Contingent claims analysis of corporate capital structures: An empirical investigation, *Journal of Finance* 39, 611-627.

Kim, Joon, Krishna Ramaswamy, and Suresh Sundaresan, 1993, Does default risk in coupons affect the valuation of corporate bonds? A contingent claims model, *Financial Management*, Special Issue on Financial Distress, Autumn, 117-131.

Lando, David, 1998, On Cox processes and credit risky securities, *Review of Derivatives Research* 2, 99-120.

Leland, Hayne E., 1994, Risky debt, bond covenants and optimal capital structure, *Journal of Finance* 49, 1213-1252.

Leland, Hayne E., and Klaus Bjerre Toft, 1996, Optimal capital structure, endogenous bankruptcy and the term structure of credit spreads, *Journal of Finance* 51, 987-1019.

Liu, Jun, Francis A. Longstaff, and Ravit Mandell, 2006, The market price of credit risk: An empirical analysis of interest rate swap spreads, forthcoming, *Journal of Business.*

Longstaff, Francis A., 1995, How much can marketability affect security values?, *Journal of Finance* 50, 1767-1774.

Longstaff, Francis A., and Eduardo S. Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance* 50, 789-819.

Longstaff, Francis A., and Eduardo S. Schwartz, 2001, Valuing American options by simulation: A simple least-squares approach, *Review of Financial Studies* 14, 113-147.

Mella-Barral, Pierre, and William Perraudin, 1997, Strategic debt service, *Journal of Finance* 52, 531-556.

Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449-470.

Nelson, Charles R., and Andrew F. Siegel, 1987, Parsimonious modeling of yield curves, *Journal of Business* 60, 473-489.

Nielsen, Lars T., Jesús Saá-Requejo, and Pedro Santa-Clara, 1993, Default risk and interest rate risk: The term structure of default spreads, Working paper, INSEAD.

Schultz, Paul H., 2001, Corporate bond trading costs and practices : a peek behind the curtain, *Journal of Finance* 56, 677-698.

Shimko, David C., Naohiko Tejima, and Donald R. Van Deventer, 1993, The pricing of risky debt when interest rates are stochastic, *Journal of Fixed Income* 3, 58-65.

Tychon, Pierre, and Vincent J. Vannetelbosch, 2005, A model of corporate bond pricing with liquidity and marketability risk, *Journal of Credit Risk*.

Wooldridge, Jeffrey M., 2001, *Econometric Analysis of Cross Section and Panel Data,* MIT Press.

Yu, Fan, 2002, Modeling expected return on defaultable bonds, *Journal of Fixed Income* 2, 69-81.

Table I
Comparative Statics of the Yield Spread Components

This table reports numerically estimated comparative statics for the perpetual debt version of the model. A ">0" or "<0" indicate a positive or negative relationship, respectively, "0" indicates no relationship, and a weak inequality sign indicates that the relationship is quantitatively weak. Note that although only one parameter is changed at a time, the default threshold is recomputed for each valuation. The benchmark parameter values employed are $\zeta = 0.10$, $\gamma = 7$, $\phi = 0.05$, $\rho = -0.5$, $C = 4$, $\sigma = 0.20$, $r = 0.05$, $\beta = 0.03$, $\alpha = 0.25$, and $d = 2$. The yield spread $s_I = s_1 + s_2 + s_3$ is decomposed as follows: $s_1 = (y_I^w - y_L^{'})$, $s_2 = (y_L^{'} - y_L^w)$, and $s_3 = (y_L^w - r)$, where y_I^w is the yield on the illiquid bond when workouts are possible, $y_L^{'}$ is the yield on a hypothetical liquid bond with identical cash flows to the illiquid bond in all states of the world, and y_L^w is the yield on a liquid bond with the same *promised* cash flows as the illiquid bond.

Panel A: High Shareholder Bargaining Power ($\eta = 0.75$)

		Total Yield Spreads ($s_1 + s_2 + s_3$)	Default Component (s_3)	Total Nondef. Component ($s_1 + s_3$)	Pure Liquidity Component (s_1)	Workout Component (s_2)
Long-run mean of liquidity shock prob.	ζ	> 0	0	> 0	> 0	> 0
Mean number of dealers	γ	< 0	0	< 0	< 0	< 0
Correlation coefficient	ρ	0	0	< 0	< 0	< 0
Leverage	C	> 0	> 0	> 0	< 0	> 0
Asset risk	σ	> 0	> 0	> 0	> 0	≥ 0
Cash flow rate	β	> 0	> 0	> 0	> 0	< 0
Liquidation costs	α	> 0	> 0	< 0	< 0	> 0
Ch. 11 duration	d	> 0	> 0	> 0	< 0	> 0

Panel B: High Bondholder Bargaining Power ($\eta = 0.25$)

		Total Yield Spreads ($s_1 + s_2 + s_3$)	Default Component (s_3)	Total Nondef. Component ($s_1 + s_3$)	Pure Liquidity Component (s_1)	Workout Component (s_2)
Long-run mean of liquidity shock prob.	ζ	> 0	0	> 0	> 0	> 0
Mean number of dealers	γ	< 0	0	< 0	< 0	< 0
Correlation coefficient	ρ	0	0	< 0	< 0	< 0
Leverage	C	> 0	> 0	> 0	< 0	> 0
Asset risk	σ	> 0	> 0	> 0	> 0	≥ 0
Cash flow rate	β	> 0	> 0	> 0	> 0	≥ 0
Liquidation costs	α	> 0	> 0	> 0	> 0	≥ 0
Ch. 11 duration	d	> 0	> 0	> 0	≥ 0	> 0

Table II
Descriptive Statistics of Bond Issues

The spread y_{it} is expressed in basis points, the maturity and age of the bonds in years, and the credit rating is based on a numerical scale ranging from 1 to 23, where 1 represents an S&P rating of AAA and 23 is the rating of a defaulted bond.

Panel A: Datastream Zero-coupon Monthly Data from January 1986 to December 1996

	y_{it} (%)	Maturity	Age	Credit Rating
Mean	0.40	12.4	5.3	1.3
Median	0.31	11.9	5.2	1.0
Maximum	10.17	29.7	16.3	8.0
Minimum	0.01	1.0	0.0	1.0
Std. Dev.	0.41	7.02	3.02	0.74
Skewness	8.02	0.28	0.64	3.24
Kurtosis	117.65	2.10	3.33	16.05

Panel B: NAIC Transaction Data from January 1996 to December 2001

	y_{it} (%)	Maturity	Age	Credit Rating
Mean	2.13	12.1	3.4	8.8
Median	1.27	8.6	2.8	9.0
Maximum	37.70	100.1	19.6	23.0
Minimum	0.20	1.0	0.0	1.0
Std. Dev	3.63	10.6	2.8	4.2
Skewness	5.9	3.6	1.1	1.5
Kurtosis	43.2	25.9	4.4	6.2

Table III
Expected Signs of Regression Variables

To proxy market volatility we use the Chicago Board Options Exchange VIX index, which is a weighted average of the implied volatilities of eight options with 30 days to maturity. We use the monthly S&P 500 return ($SPRET$) as a proxy for changes in firms' asset values. We use the difference between Moody's Baa and Aaa rated bond yield indices ($DEFPREM$) as an additional proxy for the probability of financial distress in the economy. To proxy for market liquidity, we employ $TLIQ$, the yield differential between the previous long bond and the most recently issued 30-year bond. To proxy for individual issue liquidity, we use a dummy (OTR) that indicates whether the bond was issued in the last two months. A "+" or a "−" indicate an expected positive coefficient estimate for that variable. Two signs separated by a slash (e.g. $+/++$) indicate the differences in the expectations according to the line heading (e.g., High / low rating).

	VIX	$SPRET$	$SLOPE$	r_{it}	$DEFPREM$	OTR	$TLIQ$
All	+	-	-	-	+	-	+
High / low rating	+ / + +	- / - -	- / - -	- / - -	+ / + +	- / - -	+ / + +
Short / long maturity	+	-	- / - -	-	+	- - / -	+ + / +

Table IV
Differential Impact of Liquidity Proxies in Rating and Maturity Subsamples

The results are based on the following panel regression during the period between January 1986 and December 1996:

$$
\begin{aligned}
y_{it} &= \alpha_i + \beta_1 VIX_t + \beta_2 SPRET_t + \beta_4 SLOPE_t + \beta_5 r_{it} + \beta_6 DEFPREM_t \\
&\quad + \beta_7 OTR_{it} + \beta_8 TLIQ_t + \varepsilon_{it} \\
&\quad \text{where } \varepsilon_{it} = \rho \varepsilon_{i,t-1} + \eta_{it},
\end{aligned}
$$

where VIX denotes the implied volatility index, $SPRET$ the monthly S&P 500 return, $SLOPE$ the difference between the 10- and 2-year Treasury yields, r_{it} the Treasury rate that corresponds to the maturity of the particular bond, $DEFPREM$ the difference between Moody's Baa- and Aaa-rated corporate bond yield indices. OTR is a dummy that indicates whether a given bond is on-the-run, assumed to mean less than two months of age. $TLIQ$ denotes the basis point difference in yield between the most recently issued 30-year Treasury bond and the yield on the next-most recent band. Due to the presence of serial correlation in the time series for individual bond spreads, we include an autocorrelated error structure. The first line reports the coefficient estimates and the row below the $t-$statistics. A superscript * or ** indicates significance at the 95% and 99% confidence levels, respectively. Long bonds are defined as those with a maturity exceeding 12 years. The high rating category contains all AAA-rated bonds and the lower category all the others.

	VIX	$SPRET$	$SLOPE$	r_{it}	$DEFPREM$	OTR	$TLIQ$	N	R^2
All	0.00078**	-0.00059**	-0.01644**	-0.03964**	0.25139**	–0.10313**		35476/522	6.39%
	3.85	-3.94	-4.15	-18.36	23.33	-9.56			
	0.00074**	-0.00050**	-0.01649**	-0.03724**	0.25541**		0.12422**		6.34%
	3.64	-3.33	-4.15	-16.99	23.48		5.56		
Low rating	0.00321**	-0.00198**	-0.04740*	-0.12717**	0.56290**	–0.32733**		5073/77	2.64%
	3.86	-2.78	-2.14	-12.16	10.90	-2.79			
	0.00361**	-0.00158*	-0.04411*	-0.12208**	0.56578**		0.20652*		2.39%
	3.92	-2.17	-1.97	-11.32	10.94		2.05		
High rating	-0.00061**	-0.00043**	-0.00702*	-0.01839**	0.13792**	–0.08533**		30399/448	4.56%
	-4.18	-4.18	-2.50	-11.91	18.18	-13.06			
	-0.00064**	-0.00039**	-0.00754**	-0.01703**	0.13868**		0.06031**		4.65%
	-4.38	-3.81	-2.67	-10.90	17.95		3.87		
Short maturity	0.00227**	-0.00096**	-0.01663*	-0.05881**	0.30373**	-0.14290**		17828/356	5.34%
	6.52	-3.69	-2.36	-17.03	15.50	-5.76			
	0.00225**	-0.00087**	-0.01706*	-0.05676**	0.30498**		0.09286		5.41%
	6.45	-3.30	-2.41	-16.14	15.43		1.41		
Long maturity	-0.00100**	-0.00017	-0.02687**	-0.01426**	0.17176**	-0.08639**		17516/297	12.8%
	-4.95	-1.13	-7.11	-5.90	16.67	-9.57			
	-0.00105**	-0.00008	-0.02746**	-0.01147**	0.17705**		0.13821**		12.29%
	-5.17	-0.51	-7.22	-4.67	16.99		6.20		

Table V
Differential Impact of Liquidity Proxies in Rating and Maturity Subsamples
NAIC Transaction Data 1996-2001

The results are based on the following panel regression:

$$
\begin{aligned}
y_{it} &= \alpha_i + \beta_1 VIX_t + \beta_2 SPRET_t + \beta_4 SLOPE_t + \beta_5 r_{it} + \beta_6 DEFPREM_t \\
&\quad + \beta_7 OTR_{it} + \beta_8 TLIQ_t + \varepsilon_{it} \\
&\quad \text{where } \varepsilon_{it} = \rho \varepsilon_{i,t-1} + \eta_{it},
\end{aligned}
$$

where VIX denotes the implied volatility index, $SPRET$ the monthly S&P 500 return, $SLOPE$ the difference between the 10- and 2-year Treasury yields, r_{it} the Treasury rate that corresponds to the maturity of the particular bond and $DEFPREM$ the difference between Moody's Baa- and Aaa-rated corporate bond yield indices. OTR is a dummy that indicates whether a given bond is on-the-run, and is assumed to mean less than two months of age. $TLIQ$ denotes the basis point difference in yield between the most recently issued 30-year Treasury bond and the yield on the next most recent. Due to the presence of serial correlation in the time series for individual bond spreads, we include an autocorrelated error structure following Baltagi and Wu (1999). The first line reports the coefficient estimates and the row below the $t-$statistics. A superscript * or ** indicates significance at the 95% and 99% confidence levels, respectively. Long bonds are defined as those with a maturity exceeding 12 years. The last column (N) indicates the size of the panel as the total number of observations and as the number of cross-sectional units.

	VIX	$SPRET$	$SLOPE$	r_{it}	$DEFPREM$	OTR	$TLIQ$	R^2	N
All	0.063**	0.035*	-0.189**	-0.466**	1.410**			3.50%	35983 / 1592
	16.93	2.35	-4.81	-20.74	10.44				
	0.062**	0.035*	-0.191**	-0.465**	1.384**	-0.281**		3.57%	
	16.64	2.32	-4.88	-20.66	10.23	-2.74			
	0.066**	0.031*	-0.202**	-0.470**	1.553**		-1.196	3.53%	
	17.20	2.04	-4.95	-20.64	9.33		-1.93		
	0.077**	0.040**	-0.050	-0.476**			2.149**	3.44%	
	21.27	2.62	-1.34	-20.90			4.25		

	VIX	$SPRET$	$SLOPE$	r_{it}	$DEFPREM$	OTR	$TLIQ$	R^2	N
AAA TO AA- RATING	-0.020	-0.021	-0.405**	-0.494**	0.345	-0.104		0.69%	2737 / 124
	-1.53	-0.47	-3.57	-4.76	0.78	-0.38			
	-0.018	-0.026	-0.454**	-0.449**	1.137*		-4.378*	0.87%	
	-1.33	-0.56	-3.89	-4.29	2.04		-2.31		
	-0.016	-0.024	-0.392**	-0.515**			-2.031	0.68%	
	-1.19	-0.53	-3.47	-5.21			−1.35		

A+ TO BBB- RATING	0.030**	0.019	-0.383**	-0.504**	0.702**	-0.304**		2.60%	25390 / 1143
	7.06	1.15	-9.13	-16.27	4.66	-2.95			
	0.033**	0.017	-0.401**	-0.497**	1.006**		-1.657**	2.61%	
	7.76	1.03	-9.24	-15.87	5.38		-2.51		
	0.037**	0.020	-0.322**	-0.537**			0.446	2.58%	
	8.55	1.20	-7.88	-17.59			0.84		

	VIX	$SPRET$	$SLOPE$	r_{it}	$DEFPREM$	OTR	$TLIQ$	R^2	N
BB+ to B- rating	0.159**	0.063	0.161	-0.681**	2.733**	-0.820*		10.93%	5849 / 235
	13.84	1.50	1.29	-13.02	6.64	-2.11			
	0.165**	0.058	0.183	-0.688**	2.620**		0.613	10.59%	
	14.03	1.37	1.41	-12.98	5.40		0.32		
	0.195**	0.079	0.462**	-0.632**			5.861**	10.27%	
	18.81	1.89	3.88	-12.12			3.61		
CCC+ to D rating	0.169**	-0.046	1.70739**	-1.122	3.981**	0.172		12.20%	1629 / 89
	6.27	-0.44	6.11	-0.10	4.03	0.19			
	0.179**	-0.038	1.955**	-1.173**	1.902		12.607**	12.58%	
	6.45	-0.36	6.77	-9.30	1.63		2.91		
	0.201**	-0.024	2.139**	-1.134**			16.340**	12.64%	
	8.28	-0.23	8.01	-9.14			4.43		

Table VI
The Russian Default / LTCM

The results are based on the following regression:

$$\begin{aligned} y_{it} &= \alpha_i + \beta_1 VIX_t + \beta_2 SPRET_t + \beta_4 SLOPE_t + \beta_5 r_{it} + \beta_6 DEFPREM_t \\ &\quad + \beta_7 OTR_{it} + \beta_8 TLIQ_t + \varepsilon_{it}, \\ &\quad \text{where } \varepsilon_{it} = \rho\varepsilon_{i,t-1} + \eta_{it}. \end{aligned}$$

where VIX denotes the implied volatility index, $SPRET$ the monthly S&P 500 return, $SLOPE$ the difference between the 10- and 2-year Treasury yields, r_{it} the Treasury rate that corresponds to the maturity of the particular bond and $DEFPREM$ the difference between Moody's Baa- and Aaa-rated corporate bond yield indices. OTR is a dummy that indicates whether a given bond is on-the-run, and is assumed to mean less than two months of age. We include an autocorrelated error structure following Baltagi and Wu (1999). The first line reports the coefficient estimates and the row below the $t-$statistics. A superscript * or ** indicates significance at the 95% and 99% confidence levels, respectively. Long bonds are defined as those with a maturity exceeding 12 years.

Panel A: All

	VIX	$SPRET$	$SLOPE$	r_{it}	$DEFPREM$	OTR
Run-up (January 1 1998 - August 14 1998)	-0.044	-0.095	1.531	-2.329**	7.785	-0.320
	-1.30	-0.97	1.28	-3.79	1.66	-1.11
Crisis period (August 17 1998 - November 20 1998)	-0.008	-0.073	0.668	-1.392**	-1.669	-1.191*
	-0.35	-0.94	0.72	-3.31	-0.91	-2.24
Post Crisis (November 23 1998 - October 29 1999)	0.004	-0.053	1.713	-1.207**	0.150	-0.495*
	0.22	-1.26	1.44	-6.63	0.13	-1.98

Panel B: Investment grade

	VIX	$SPRET$	$SLOPE$	r_{it}	$DEFPREM$	OTR
Run-up (January 1 1998 - August 14 1998)	-8.196	-0.160	1.952	-2.321**	10.126	-0.376
	-2.16*	-1.45	1.47	-3.40	1.93	-1.20
Crisis period (August 17 1998 - November 20 1998)	0.577	-0.085	0.675	-1.255**	-1.273	-0.999
	0.22	-1.00	0.66	-2.73	-0.63	-1.77
Post Crisis (November 23 1998 - October 29 1999)	-0.038	-0.007	2.450	-1.169**	1.119	-0.427
	-0.02	-0.15	1.84	-5.82	0.87	-1.59

Panel C: BB+ to D

	VIX	$SPRET$	$SLOPE$	r_{it}	$DEFPREM$	OTR
Run-up (January 1 1998 - August 14 1998)	9.146	0.161	0.094	-2.174	-0.807	-0.187
	1.21	0.75	0.03	-1.58	-0.08	-0.27
Crisis period (August 17 1998 - November 20 1998)	-4.864	-0.048	0.335	-1.598	-2.727	-1.582
	-0.70	-0.27	0.13	-1.22	-0.72	-0.81
Post Crisis (November 23 1998 - October 29 1999)	2.416	-0.200	-0.857	-1.099**	-2.836	-0.366
	0.59	-2.15	-0.33	-2.73	-1.11	-0.64

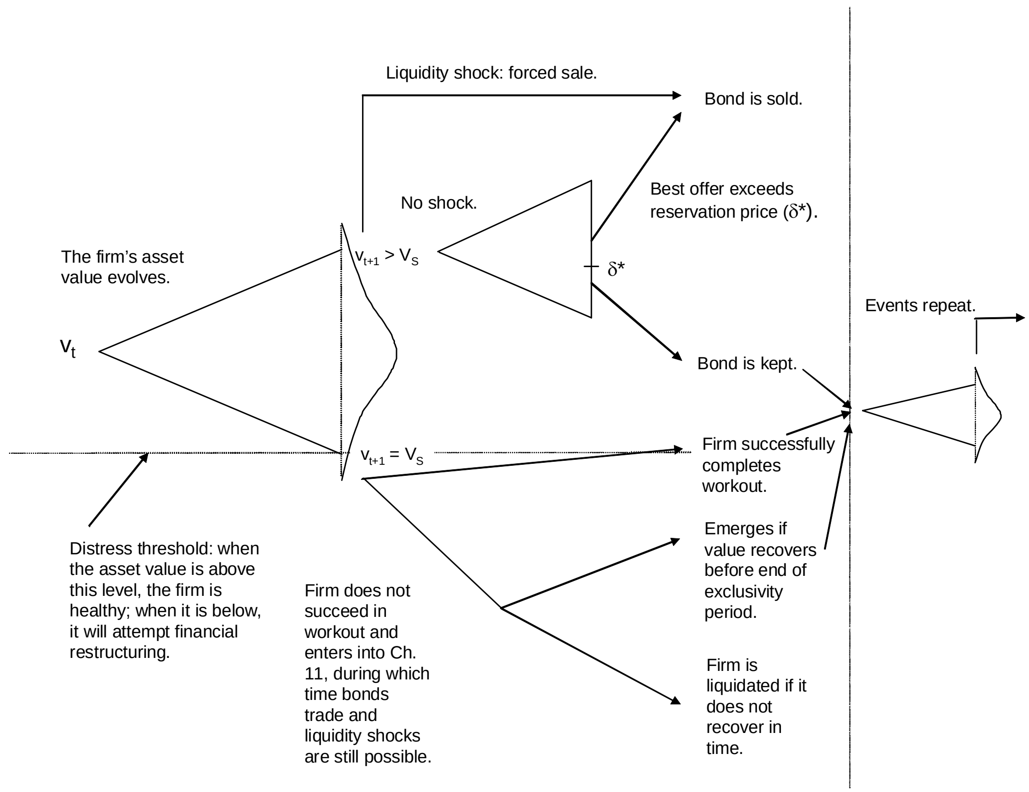

Figure 1: The sequence of events.

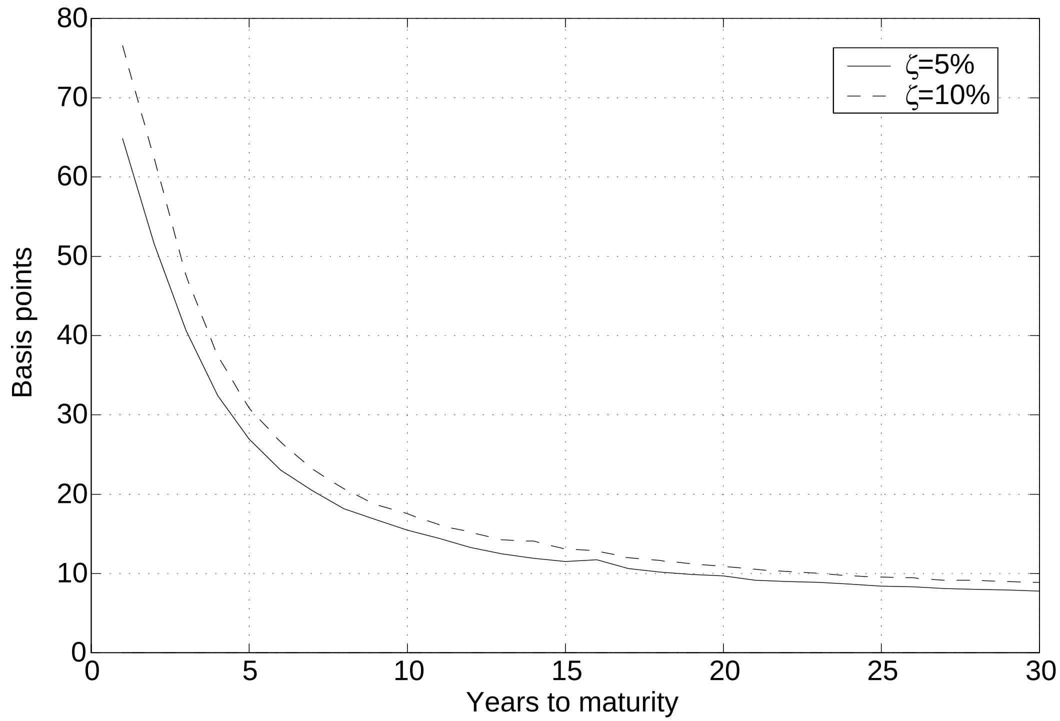

Figure 2. The illiquidity spread and the annualized probability of a liquidity shock - no default risk. The $y-$ axis measures the yield spread in basis points and the $x-$ axis the time to maturity in years for individual bonds. Parameter values: $r = 0.05$, $\Delta t = 1/12$, $\gamma = 7$, $\phi = 0.05$ and $\kappa = 0.5$. Notation: r is the risk free rate, Δt the time step, γ the mean number of active dealers, ϕ the volatility parameter of the instantaneous liquidity shock probabilities λ_t, and κ the mean reversion speed of λ_t. Long run mean probabilities of a liquidity shock: $\zeta = 0.05$ (solid line) and $\zeta = 0.1$ (dashed line) with $\lambda_0 = \zeta$.

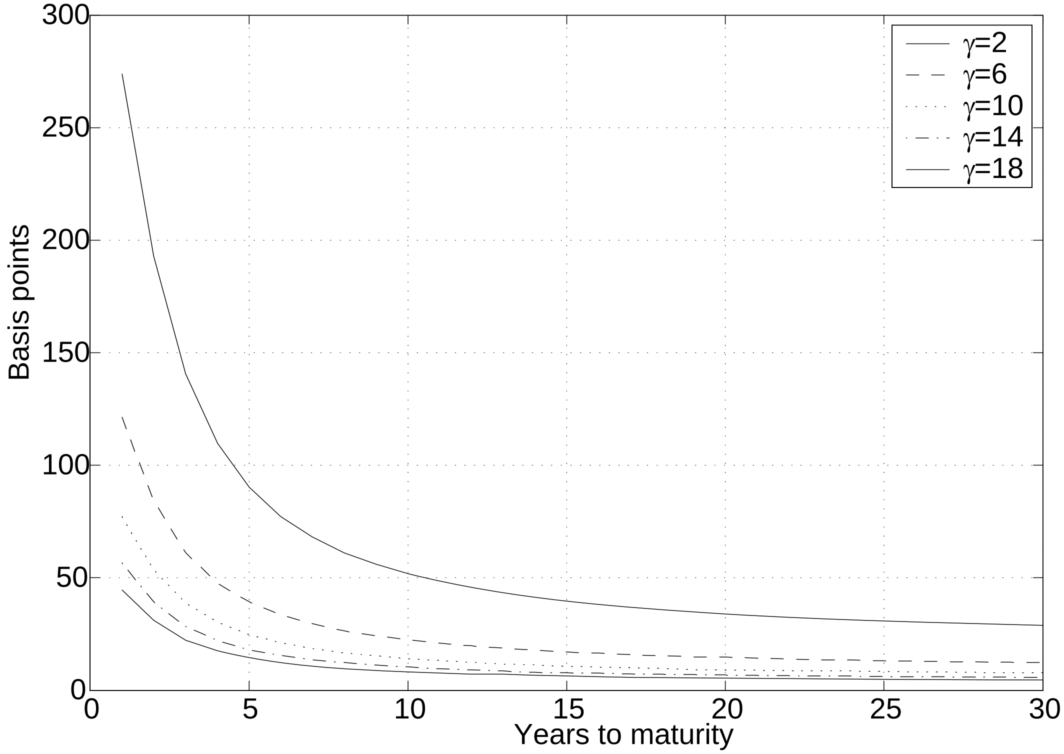

Figure 3. **The illiquidity spread and the mean number of active dealers - with default risk.** The $y-$ axis measures the yield spread in basis points and the $x-$ axis the time to maturity in years for individual bonds. The maturity of newly issued debt is 30 years. Parameter values: $r = 0.05,\ \beta = 0.03,\ d = 2,\ \Delta t = 1/12,\ C = 4, P = 80,\ \sigma = 0.20,\ \alpha = 0.25,\ \eta = 0.5,\ \phi = 0.05,\ \zeta = 0.1,\ \rho = -0.5$, and $\kappa = 0.5$. Notation: r is the risk free rate, Δt the time step, γ the mean number of active dealers, ϕ the volatility parameter of the instantaneous liquidity shock probabilities λ_t, ρ the instantaneous correlation between asset value v_t and λ_t, and κ the mean reversion speed of λ_t. We set λ_0 equal to ζ, the long run mean instantaneous probability of a liquidity shock.

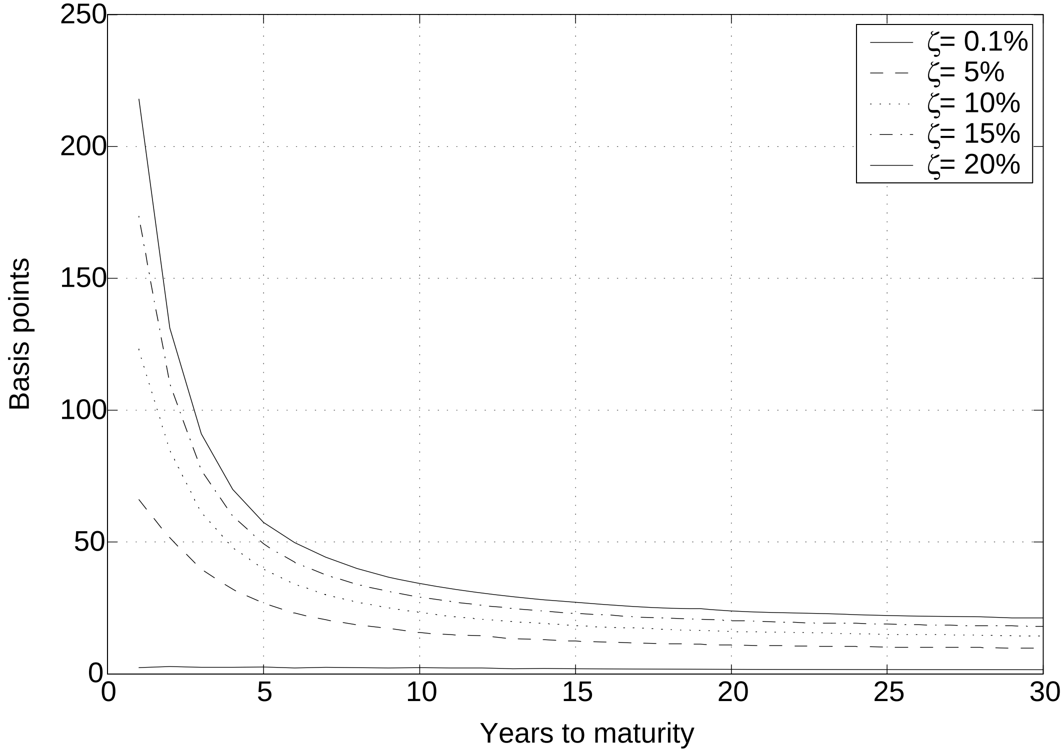

Figure 4. The illiquidity spread and the annualized probability of a liquidity shock - with default risk. The $y-$ axis measures the yield spread in basis points and the $x-$ axis the time to maturity in years for individual bonds. The maturity of newly issued debt is 30 years. Parameter values: $r = 0.05,\ \beta = 0.03,\ d = 2,\ \Delta t = 1/12,\ C = 4,\ P = 80, \sigma = 0.20,\ \alpha = 0.25,\ \eta = 0.5,\ \gamma = 7,\ \phi = 0.05,\ \rho = -0.5$, and $\kappa = 0.5$. The maturity of newly issued debt is 30 years. Notation: r is the risk free rate, Δt the time step, γ the mean number of active dealers, ϕ the volatility parameter of the instantaneous liquidity shock probabilities λ_t, ρ the instantaneous correlation between asset value v_t and λ_t, and κ the mean reversion speed of λ_t. We set λ_0 equal to ζ, the long run mean instantaneous probability of a liquidity shock.

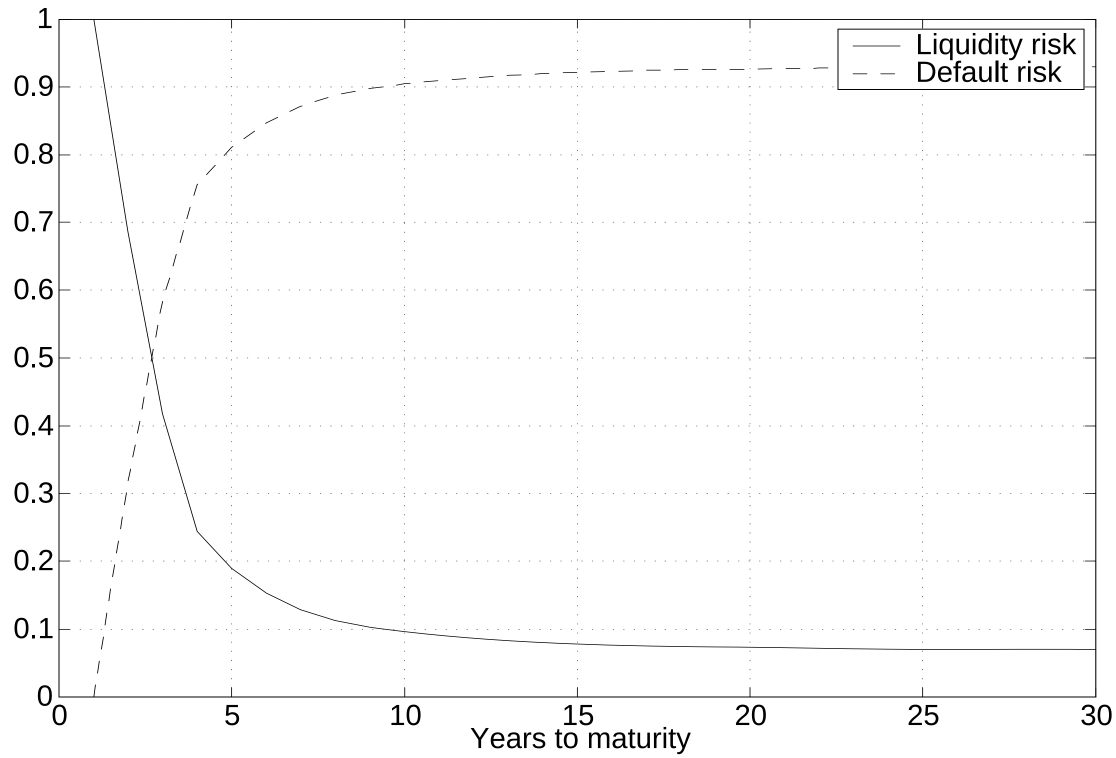

Figure 5. The relative size of the default and nondefault components. The figure shows the following ratios: $\frac{s_1+s_2}{s_1+s_2+s_3}$ (liquidity risk) and $\frac{s_3}{s_1+s_2+s_3}$ (default risk). The component s_1 measures the impact on the value of possible liquidity shocks while the firm is solvent, s_2 measures the impact of illiquidity on bargaining given in distress, and s_3 measures the default component of the yield spread. The $x-$ axis represents the time to maturity in years for individual bonds. The maturity of newly issued debt is 30 years. Parameter values: $r = 0.05,\ \beta = 0.03,\ d = 2,\ \Delta t = 1/12,\ C = 4,\ P = 80,\ \sigma = 0.20,\ \alpha = 0.25,\ \eta = 0.5,\ \gamma = 7,\ \zeta = 0.10,\ \phi = 0.05,\ \rho = -0.5$, and $\kappa = 0.5$. Notation: r is the risk free rate, Δt the time step, γ the mean number of active dealers, ϕ the volatility parameter of the instantaneous liquidity shock probabilities λ_t, ρ the instantaneous correlation between asset value v_t and λ_t, and κ the mean reversion speed of λ_t. We set λ_0 equal to ζ, the long run mean instantaneous probability of a liquidity shock.