Quantifying Risk in the Electricity Business: A RAROC-based Approach

M.Prokopczuk, S.Rachev and S.Trück

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Abstract

The liberalization of electricity markets has forced the energy producing companies to react to the new situation. The abolishment of monopolies and the launch of open markets have increased the need of calculating costs closer to the profit frontier to be still competitive, not only against the other German but also against foreign suppliers. Thus, an efficient risk management and risk controlling are needed to ensure the financial survival of the company even during bad times. In this work we use the RAROC methodology to develop a Monte Carlo Simulation based model to quantify risks related to wholesale electricity contracts, also called full load contracts. We do not only consider risk due to market price fluctuations but also due to correlation effects between the spot market price and the load curve of a single customer.

Stefan Trück[1], Marcel Prokopczuk
Email: stefan@statistik.uni-karlsruhe.de, marcel@prokopczuk.de
Institut für Statistik und Mathematische Wirtschaftstheorie
Universität Karlsruhe
Kollegium am Schloss, D-76128 Karlsruhe, Germany
Fon: +49 (0) 721 608 8113, Fax: +49 (0) 721 608 3811

Svetlozar T. Rachev[2]
Email: rachev@pstat.ucsb.edu
Institut für Statistik und Mathematische Wirtschaftstheorie
Universität Karlsruhe
Kollegium am Schloss, D-76128 Karlsruhe, Germany
and
Department of Statistics and Applied Probability
University of California, Santa Barbara, CA 93106, USA

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[2]Rachev gratefully acknowledge research support by grants from Division of Mathematical, Life and Physical Sciences, College of Letters and Science, University of California, Santa Barbara, the Deutschen Forschungsgemeinschaft and the Deutscher Akademischer Austausch Dienst.

1 Introduction

In 1997 the EU directive 96/92/EC started the deregulation process of European electricity markets with the goal of achieving a more efficient supply with electricity in competitive markets. In Germany this directive was implemented through the amendment of the power industry law in 1998. This liberalization led to the establishment of a regulated electricity market, which institutionalized itself in two electricity exchanges, the EEX in Frankfurt and the LPX in Leipzig. In the year 2002 both exchanges merged to bundle market activity.

Electricity companies were forced to deal with the new situation. In reaction, energy trading companies were established, which should act as intermediate between the power generating and the sales businesses, as well as with the outside market. These trading companies are responsible of capturing and evaluating the risks occurring when electricity is traded at a spot and future market.

The exchange itself is not the main distribution channel, since most customers do not want to bother to buy electricity at an exchange. They rather make direct contracts with the electricity company to provide them with electricity for a fixed price per unit.

Entering such a contract, also called full load contract, the electricity trader commits himself to the obligation to deliver electricity for a fixed price. This means the trader is willing to bear several kinds of risks in place of the customer, for which the trader should be compensated. In the following work we want to quantify the risk related to full load contracts. Furthermore we want distinguish between customers according to their load profiles, which are a main determinant of the riskiness of the contract.

2 Introduction to the RAROC Framework

2.1 Performance Measures without Risk

The traditional performance measures to evaluate the performance of a company, business unit or single investment are mainly RoI - Return on Investment and RoE - Return on Equity. RoI compares the return to the amount of invested money, where RoE only takes the invested equity capital into account. Written as formulas we get:

$$\text{RoI} = \frac{\text{Return}}{\text{Invested Capital}} \tag{2.1}$$

and

$$\text{RoE} = \frac{\text{Return}}{\text{Invested Equity Capital}} \tag{2.2}$$

The shortcomings of these concepts are obvious: They are accounting-based and do not reflect the real performance[3]. Neither do they take risk into account nor is it possible to determine the denominator for single business units from the firm's balance sheet.

The point that RoI, RoE and similar measures do not take risk into considerations is a big problem. Suppose there are two investments, both have the same return rate but one of them is much riskier. If you compare the two investments they would have the same RoI or RoE and would be regarded as equal. Obviously this is not true. Since banks are usually considered being risk-averse, they would prefer the less risky investment and for taking more risk they require a premium, i.e. a higher return.

As a consequence the return has to be compared to the risk undertaken, otherwise it would be impossible to compare the performances of two different investments or business units, e.g. the trading desks for derivatives and government bonds. There are many measures to achieve this. In the next section we present the most popular ones.

2.2 Performance Measures for risky Portfolios

The need to compare the performance of portfolios and business units with respect to their risk is not new. Based on the portfolio and capital market theory several measures have been developed (see (PS99), p.295ff). The most popular ones are:

Jensens Alpha

Based on the capital market line of the CAPM[4], Jensen assumes it is possible to gain profit out of a disequilibrium in the market. If single assets are over- or underpriced and the portfolio manager recognizes this, he can utilize it and go short or long in this position, respectively.

Jensens Alpha measures the difference between the actual rate of excess return and the theoretical one given by the CAPM, i.e.:

$$\alpha = (r - r_f) - (\mathbb{E}[r] - r_f) \tag{2.3}$$

where r is the portfolio-return and r_f the risk free interest rate. Plugging in the CAPM equation $\mathbb{E}[r] = r_f + \beta(\mathbb{E}[r_m] - r_f)$ and evaluating ex-post this becomes:

$$\alpha = (r - r_f) - [\beta(r_m - r_f)] \tag{2.4}$$

where r_m is the return of the market portfolio and β the systematic risk of the investment (for details on the CAPM see for example (PS99)).

[3]In banking the uselessness of RoE is especially high, because many projects are completely financed with debt capital, thus have an infinite RoE.

[4]Capital Asset Pricing Model

Jensens Alpha measures the performance of the portfolio compared to the market and thus makes it possible to compare two portfolio managers. But since it takes only the systematic risk into account, this comparison is only fair if two portfolios have the same systematic risk, which is not true in general.

The Treynor Ratio

The Treynor ratio, also called reward-to-volatility-ratio, measures the excess return adjusted by the systematic risk. Thus it is subject to the same criticism as Jensens Alpha. It is given by:

$$T = \frac{(\mathbb{E}[r] - r_f)}{\beta} \tag{2.5}$$

The Sharpe Ratio

The reward-to-variability-ratio of Sharpe is similar to the Treynor ratio but adjusts the excess return with the overall risk, i.e. systematic and unsystematic risk, measured by the standard deviation σ of the portfolio:

$$S = \frac{(\mathbb{E}[r] - r_f)}{\sigma} \tag{2.6}$$

The Sharpe Ratio has the advantage that it takes also unsystematic risk into account, i.e. it can be used to compare undiversified portfolios. On the other hand, bank portfolios can be usually regarded as well diversified, so this advantage does not really count.

The general problem of these measures is that they lead to dimensionless numbers, which are well suited to compare single portfolios, but do not enable the management to control the overall risk of the firm. Furthermore Jensens Alpha and the Treynor Ratio are based on the CAPM and thus also subject to the criticism of it. The Sharpe Ratio does not have this problem but when considering Risk Management the standard deviation does not seem to be the appropriate risk measure. Risk Management aims to protect the company from heavy downward movements, i.e. big losses, but the standard deviation is also sensible to upward movements.

3 Risk Adjusted Performance Measures

In the need of an efficient Risk Management and the ability to compare different business units new Risk Adjusted Performance Measures (RAPMs) have become popular in the banking business. Many acronyms for RAPMs can be found in the literature, e.g. RAR, ROC, RAROC, RORAC, RARORC, RAROEC, RARORAC. This can be very confusing, especially because same acronyms can stand for different things and equal things sometimes have different names.

The confusion about the naming of RAPMs is basically a result of the historical development. In the late 1970s Bankers Trust developed a RAPM to measure credit risk and called it RAROC - Risk adjusted Return on Capital. This was defined as:

$$RAROC_{BankersTrust} = \frac{\text{Risk-adjusted Return}}{\text{Equity Capital}} \tag{3.1}$$

What is the "Risk-adjusted Return"? Bankers Trust wanted to include in their calculations the fact that it is possible that a debtor defaults, i.e. he does not pay back the loan. Thus, they subtracted the expected loss from the deterministic return receiving for the loan. However, the name "Risk-adjusted Return" is misleading. Since the expected loss is known, there is no risk involved. If, for example, the credit debtor of a 5 years loan has a S&P[5]-Rating of BBB, which corresponds to an average default rate of 2.1% (see (Jor01) p.319), the expected loss of a loan of 1,000,000 Euro would be 21,000 Euro. With an interest rate of 10% on the loan, the Expected Return can be computed as: $0.1 \cdot 1,000,000$ Euro $-$ $21,000$ Euro $=$ $79,000$ Euro. Thus, the "Risk-adjusted Return" is in reality just the Expected Return of the business.

In the denominator, Bankers Trust used the Equity Capital involved. As described in section 2.1.1 this has several shortcomings, why nowadays not the Equity Capital, but the Economic Capital is used. Thus, the RAROC is defined as:

$$RAROC = \frac{\text{Expected Return}}{\text{Economic Capital}} \tag{3.2}$$

The Economic Capital (EC) is neither the required regulatory capital[6] for the business nor does it correspond to the Equity Capital used. The EC is the amount of money which is needed to secure the banks survival in a worst case scenario, i.e. it is a buffer against heavy shocks. It should capture all types of risk (market, credit and operational risk) and is often calculated by VaR - the Value at Risk. The VaR is a quantile of the profit and loss (P&L) distribution, i.e. it measures the maximum amount of money one can lose at a given confidence level in a specified period of time. If X is the random variable describing the profit and loss of the business, the formula to compute VaR at a level of α is:

$$P(X < -VaR) = \alpha \tag{3.3}$$

what means

[5] Standard & Poors, American Rating Company

[6] In Germany determined by the KWG (Kreditwesengestz) and the Grundsatz 1.

$$\int_{-\infty}^{-VaR} f(x)dx = \alpha \tag{3.4}$$

where $f(x)$ is the density of the profit and loss distribution. This means you are looking for the alpha-quantile of the P&L-function. Figure 1 shows for example the VaR at the five percent level of a given P&L distribution. (More details about VaR can be found in (Jor01) or (Dow98)).

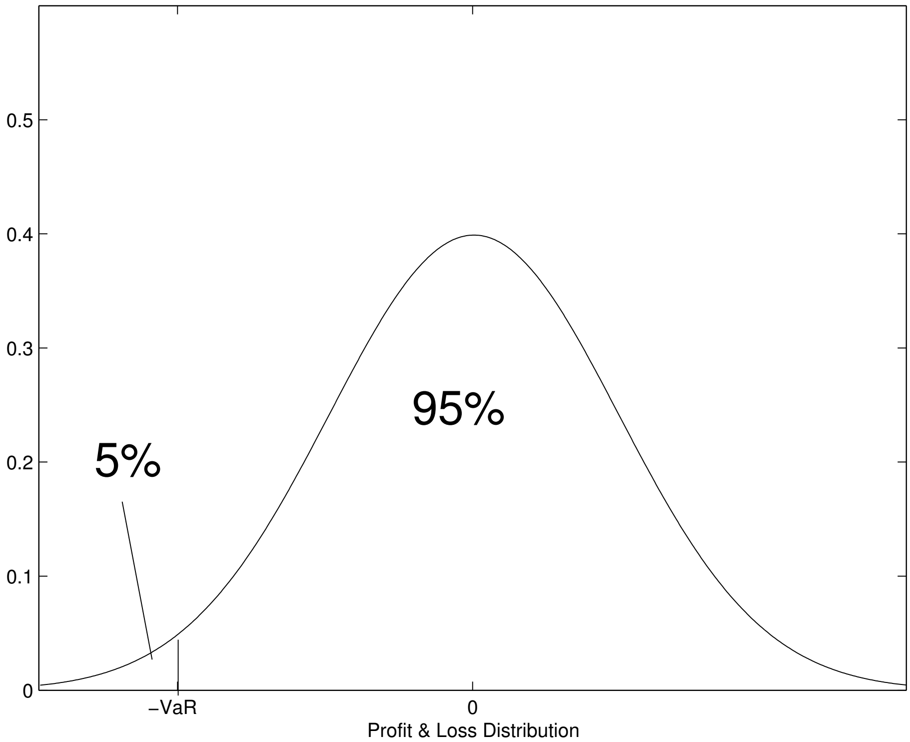

Figure 1: Value at Risk

If we express the Economic Capital as VaR, (3.2) becomes:

$$RAROC = \frac{\text{Expected Return}}{\text{VaR}} \tag{3.5}$$

The appealing thing about RAROC is that it provides a uniform measure of performance that the management can use to compare businesses with different sources of risk and capital requirements (ZWKJ96). Especially the ability to calculate the Economic Capital became more and more important in the past, since the view that Equity Capital is a scarce resource has become the general opinion. If the sum over the ECs of all possible investments of a firm is higher than its Equity Capital, the management has to decide which of those investments should be made[7]. Hence, RAROC is not only suited to compare all kinds of businesses with each other, it is also a powerful management tool for capital allocation and risk control.

4 RAROC and EVA

4.1 Shareholder Value and Economic Profit

We stated that RAROC is able to compare two investments A and B and to decide which of them is the better one. But to know that A is better than B does not necessary mean that doing this business is profitable for the company. Since the ultimate goal of a company is to increase its Shareholder Value, the decision rule in a RAROC framework should be:

$$\text{Invest in project A} \iff \text{RAROC(A)} > \mu \tag{4.1}$$

The question to be asked here now is: What is the so called hurdle rate μ?

If, as described before, your priority lies on the Shareholder Value, your goal must be to have a higher RAROC than the Cost of Equity Capital. Note that from the accounting point of view, the cost of Equity Capital is neglected. But if you want to decide whether an investment adds Shareholder Value, i.e. generates a real economic profit, this opportunity costs have to be considered. This concept is often referred to as EVA[8] - Economic Value Added. Stated in one formula it means:

$$\text{Increase in Shareholder Value} \Longleftrightarrow \text{RAROC} > \text{Cost of Equity Capital Rate} \tag{4.2}$$

Note that some authors prefer to subtract the Cost of Equity Capital already from the Expected Return in the RAROC formula, then (3.5) and (4.2) become:

$$RAROC = \frac{\text{Expected Return - (Cost of Capital Rate} \cdot \text{VaR)}}{\text{VaR}} \tag{4.3}$$

and

$$\text{Increase in Shareholder Value} \Longleftrightarrow \text{RAROC} > 0 \tag{4.4}$$

So far, we have just renamed the hurdle rate with Cost of Equity Capital Rate.

[7] Of course diversification effects should be taken into consideration.
[8] EVA is a registered trademark of Stern Stewart & Co

Not much is gained except the fact that we now have an idea by what the hurdle rate is affected. Determining the Cost of Equity Capital is not easy and there can be found several approaches to this problem. One possible way to address this question is just to ask what rate of return the shareholders expect from their investment? This expected return rate could be our internal hurdle rate. Another idea is to let the management decide what return they want to achieve. Both ways are not really scientifically satisfying but used in practice.

4.2 Determining the Cost of Equity Capital

To determine the expected rate of return of the shareholders in a scientifically founded way, basically, two approaches could be used: The Capital Asset Pricing Model (CAPM) if we assume frictionless capital markets and the model of Froot and Stein (see (FS98)) if we drop this assumption.

Cost of Capital with the CAPM

Using the CAPM, one can argue that, if an investor is well diversified, he only wants a premium for taking over systematic risk, i.e. risk related to the entire market. Thus we can use the basic CAPM equation to determine the expected return of an investor in a company i:

$$\mathbb{E}[r_i] = r_f + (\mathbb{E}[r_m] - r_f) \cdot \beta_i \tag{4.5}$$

where
r_i = return of company i
r_f = risk-free rate of return
r_m = return of the market portfolio[9]
$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}$

This model is appropriate if the assumptions of a frictionless capital market and a perfect diversified investor holds. Relaxing these assumptions the debt to equity ratio becomes important to determine the cost of Equity Capital. In this case one can use Option Pricing Models (OPMs) as described in the next paragraph.

Cost of Capital with OPMs

Option Pricing Theory can be used to determine the value of the Equity Capital, which can be interpreted as a call option on the total firm capital. Raising debt capital can be seen as selling of assets of the company with the agreement to be allowed to continue using them, and at the same time, selling a call option with the right to buy the assets at the maturity date of the loan back for the amount of the loan.

[9]Usually an index is used as market portfolio, for example DAX or MDAX in Germany or S&P500 in the USA.

To compute the value of this option we can use the pricing formula for European Calls of Black and Scholes:

$$c = K \cdot N\left(\frac{\ln(\frac{K}{X}) + [r_f + (\sigma^2/2)]t}{\sigma\sqrt{t}}\right) - e^{-r_f t} \cdot X \cdot N\left(\frac{\ln(\frac{K}{X}) + [r_f - (\sigma^2/2)]t}{\sigma\sqrt{t}}\right) \tag{4.6}$$

with
c = price of the call option
K = today's price of the underlying
X = strike price
$N(\cdot)$ = distribution function of the gaussian distribution
σ^2 = volatility of the underlying

If we plug in the value of the entire company V as underlying and the amount of dept capital DC as strike price we have a formula for the market value of the Equity Capital EqC:

$$EqC = V \cdot N\left(\frac{\ln(\frac{V}{DC}) + [r_f + (\sigma^2/2)]t}{\sigma\sqrt{t}}\right) - e^{-r_f t} \cdot DC \cdot N\left(\frac{\ln(\frac{V}{DC}) + [r_f - (\sigma^2/2)]t}{\sigma\sqrt{t}}\right) \tag{4.7}$$

Using the CAPM and the average return of investments $\overline{r}$, one can derive the Cost of Equity Capital (see (PS99) for details):

$$r_{EqC} = r_f + N\left(\frac{\ln(\frac{V}{DC}) + [r_f + (\sigma^2/2)]t}{\sigma\sqrt{t}}\right) \cdot (\overline{r} - r_f) \cdot \frac{V}{EqC} \tag{4.8}$$

As (PS99) point out, the practical use of this equation is very limited, since again strong assumptions had to be made and the determination of several parameters is empirically almost impossible.

The model of Froot and Stein

Froot and Stein developed in 1998 a model which allows for the existence of friction in the capital market (FS98). They assume that a bank, with an existing portfolio, has the choice either to accept or reject a new loan, whose size is small compared to the entire portfolio. They decompose the risk of the loan into tradable and non-tradable components. To price the tradable risk, they use the CAPM, the non-tradable risk depends on the bank's level of risk aversion. Above all, they show that, if a bank is risk-averse, it will always hedge its tradable risk because nothing can be earned by taking over this kind of risk. The hurdle rate μ developed in Froot and Stein's model is given by:

$$\mu = g \cdot Cov(\varepsilon_T, \varepsilon_m) + G \cdot Cov(\varepsilon_N, \varepsilon_P) \tag{4.9}$$

where
g = market price of risk
G = the bank's level of risk aversion
ε_T = the tradable risk of the loan
ε_N = the non-tradable risk of the loan
ε_m = the systematic market risk factor
ε_P = the non-tradable risk of the entire portfolio

Like the other approaches this model is more of theoretical than of practical use. The assumptions made are more realistic compared to the CAPM but even more unknown variables (e.g. the risk aversion of the bank) enter the calculation. Nevertheless Froot and Stein achieve interesting results as the point that the existence of banks can be justified by their assumption of non-tradable risk.

4.3 Hurdle Rates for different Business Units

While determining the hurdle rate, one hits the question whether there should be different hurdle rates for each business unit, which would be consistent with modern finance theory because only the systematic risk should be considered which could be different for each unit. The Economic Capital always reflects the systematic and the unsystematic risk, but since a bank can be regarded as well diversified only the systematic risk is important. Because of this, each business unit has to be assigned with its own beta which then can be used to compute its hurdle rate, using for example (4.5).

On the other hand it seems desirable to have a corporate wide identical hurdle rate which reflects the ambition of the investors and the management. More practical arguments for one uniform hurdle rate are the difficulties in estimating the betas for each business unit and most of all the influence costs coming up from internal "fights" when the hurdle rates are determined. This is why many banks, e.g. the Bank of America, use one corporate wide hurdle rate (see (ZWKJ96)).

4.4 Ex-ante vs. Ex-post RAROC

As described in previous sections, RAROC can be basically used with two different intentions:

1. Performance Measurement (ex-post)
2. Capital Allocation (ex-ante)

The latter are forward looking decisions, usually based on the historical performance. When using RAROC as ex-post performance measure it must be decided whether the allocated (ex-ante view) or the actually utilized (ex-post view) Economic Capital should be used. Why is this a problem? On the one hand, one can argue that only the utilized capital should be considered, because otherwise the incentive to use all capital even if it does not make sense could be created. On the other hand, if only the utilized capital were used to measure the performance of a business unit, there could be a trend to underinvestment since the manager would not want to decrease his RAROC. This could happen if the project's RAROC is lower than the business unit's average RAROC but high enough to add Shareholder Value (i.e. the Economic Profit is higher than zero). Obviously this is not desirable, especially because capital is a scarce resource which must not be wasted.

To get an answer to this problem one should consider how the capital allocation process works:
If it is a top-down process, i.e. the top management decides about the capital allocation to each business unit without consulting the units themselves, only the utilized capital should enter the performance evaluation. The business units should not be punished for the potentially bad management decisions to assign too much capital to them.

If the capital allocation is decided on request of the business units, i.e. they have to apply for the capital, they should be punished if their requests were too high and thus, the allocated capital should be used to evaluate their performance.

This procedure is still far from perfect, since usually a mixture of these two capital allocation processes will be used. (Sai99) proposes a intermediate solution: The utilized capital is used to calculate the RAROC and a penalty on unutilized capital is subtracted. This penalty rate will be somewhere between zero and the hurdle rate. Depending on the level of the penalty rate the mentioned problems can be addressed more or less.

In this chapter we introduced the general RAROC concept, how it is used in banking and discussed some of the issues arising when applying it in practice. However, the focus of this thesis is on the energy business and not banking. Therefore, the following chapter will give an overview over the German Energy Market before developing an Energy-RAROC model.

5 The SMaPS Model

In this section we introduce the model we will use later to simulate trajectories for the spot price process. This model, called SMaPS (Spot Market Price Simulation) was developed by EnBW Gesellschaft für Stromhandel mbH, section Risk Controlling and the University of Karlsruhe (TH) (see (BKMS04)). In the first section we describe the model, in the second section we shortly discuss advantages and disadvantages of it.

5.1 Model Description

The SMaPS model is a three factor model in discrete time with hours as time unit. It is based on three different stochastic processes, which are assumed to be independent of each other :

- a load process $(L_t)_{t\in\mathbb{Z}_+}$
- a short term market process $(X_t)_{t\in\mathbb{Z}_+}$
- a long term process $(Y_t)_{t\in\mathbb{Z}_+}$

The fundamental model equation is:

$$S_t = exp(f(t, \frac{L_t}{v_t}) + X_t + Y_t) \tag{5.1}$$

where $f(t,\cdot)$, $t \in \mathbb{Z}_+$ is the so-called price-load curve (PLC) and ν_t is the average relative availability of power plants.

The load process L_t describes the demand of electricity in each hour t. This load can be directly observed and thus, estimating with historical data is possible without considering the spot prices. (BKMS04) model the load process as sum of the deterministic load forecast $\hat{L}_t$ and a SARIMA time series model with a lag of 24h L'_t:

$$L_t = \hat{L}_t + L'_t \tag{5.2}$$

The deterministic quantity $\nu_t \in [0,1]$ denotes the relative availability of power plants on the market we want to generate price paths (1 stands for full availability). Our focus is on the German market, where the availability in summer is lower than in the winter because maintenance work is conducted then. (BKMS04) refer to the quantity L_t/ν_t as adjusted load, showing in their statistical analysis that using the adjusted load leads to more realistic results than using just the load L_t.

The PLC $f : \mathbb{Z}_+ \times [0,\infty) \to \mathbb{R}$ describes the nonlinear relationship between the adjusted load and the spot price. Because it depends on many uncertain parameters, (BKMS04) decided to use an empirical estimate of this function from historical load and price data. The authors also show that the PLC differs over time, especially it changes from weekdays to weekends and peak hours to offpeak hours. Thus they fit different PLC for different weekdays and daytimes.

The market process X_t models the short term behavior of the market. The market price fluctuations are due to an effect, (BKMS04) call the "psychology of the market". They also include outages of power plants in the process X_t. As L_t, also X_t is modelled as SARIMA model with a seasonality of 24h.

The long term process Y_t reflects the stochastic nature of future prices. It is modelled as random walk with drift and include the information given by future prices from the market into the model.

Having defined the basic components of the SMaPS model we now want to describe the model selection and fit of (BKMS04). As first step they determine the empirical PLC of each hour of the day and each day of the week by fitting cubic splines to the historical data. Because the three stochastic processes are assumed to be independent of each other ((BKMS04) justify this assumption by statistical analysis) they can be modelled one by one. For the stochastic component of the load process L'_t, a SARIMA(1,0,1)x(1,0,1) model is selected. The parameters are estimated using standard maximum-likelihood estimators. For the short term market process and the long term process, the authors describe the problem that the spot price is a function of both of them. They suggest two different solutions for this problem: First one can assume $Y_t \equiv 0$ for the historical data. Then one can model and calibrate X_t using historical data with the relationship $X_t = ln(S_t) - f(t, L_t/\nu_t)$. Another approach to this problem is the usage of the so-called Kalmann filter. The authors describe this mathematical technique but also show that it does not lead to significantly different results. Thus they use the more simple approach by setting $Y_t \equiv 0$ to calibrate the short term process. A SARIMA(1,0,1)x(1,0,1) model is fitted for X_t.

The long term process is modelled as a random walk with drift:

$$Y_{t+1} = Y_t + (\mu - \frac{1}{2}\sigma_Y^2) + \sigma_Y \varepsilon_t^Y \tag{5.3}$$

where ε_t^Y are independent normally distributed random variables. Even though the original model is in discrete time, the authors use the continuous time extension to derive their results which can be used as approximation for the discrete setting. The continuous version of (5.3) is given by a Brownian Motion:

$$dY_t = (\mu_t - \frac{1}{2}\sigma_Y^2)dt + \sigma_Y dW_t \tag{5.4}$$

The authors now switch to an equivalent martingale measure P^* to estimate the parameters assuming a zero market price of risk for L_t and X_t[10]. Under the measure P^* (5.4) becomes

$$dY_t = (\mu_t - \lambda_t - \frac{1}{2}\sigma_Y^2)dt + \sigma_Y dW_t \tag{5.5}$$

where λ_t denotes the market price of risk for Y_t. Hence, the growth rate μ_t^* in the risk neutral world is

$$\mu_t^* = \mu_t - \lambda_t \tag{5.6}$$

(BKMS04) derive a formula for μ_t^* using the fact that the distributions of X_t and L_t can be approximated with their stationary distributions if the delivery

[10]See the next section for a justification of this assumption.

period is far enough in the future. The derived formula for μ_t^* is:

$$\mu_T^* = \frac{\partial}{\partial T}\left(log\frac{F_{t,T}}{\hat{S}_T}\right) \tag{5.7}$$

where $F_{t,T}$ is the price of a future at time t with delivery hour T and $\hat{S}_t$ is given by:

$$\hat{S}_t = exp(Var[X_t]/2)\mathbb{E}[exp(f(T, L_T/\nu_T))] \tag{5.8}$$

Using the SMaPS model we are now able to generate trajectories of the spot price process in the risk neutral world and use the risk neutral pricing approach for evaluating contracts written on the spot price as underlying. We will denote the expectation under the risk neutral measure with $\mathbb{E}^*[\cdot]$ throughout the entire thesis.

For illustration figure 2 shows one spot price trajectory generated with the SMaPS model. It is worth noticing that the distribution of the generated spot prices fulfills the requirement of heavy tails (leptocurticity), i.e. it has more mass in the tails than the normal distribution, which can be also observed in the historical spot price time series[11].

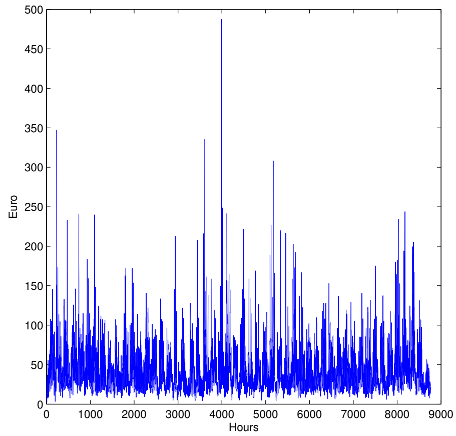

Figure 2: Trajectory generated with the SMaPS model

5.2 Model Discussion

The assumption of zero market price of risk may seem a bit over simplifying. However, this approach is not unusual for electricity spot price models. An economic justification for this assumption can be found in (Hul03), chapter 29.

[11]For an empirical analysis see for example (Dei03).

Since the changes in electricity prices and the return on the market are very weakly correlated, it can be reasonable to assume a zero market price of risk what is equivalent to assume that electricity prices behave the same way in both, the real and the risk-neutral world. (Hul03) concludes this reasoning: "The parameters [...] can therefore be estimated from historical data."

We are aware of the fact that even if it can be justified as mentioned above, the assumption of zero market price of risk is theoretically not completely correct. The correlation between the changes of electricity prices and returns on the stock market are low, but not zero. Nevertheless, we decided to use the SMaPS model due to several advantages.

First, compared to other models, it does not only model the spot price using the spot price history, it also takes the grid load into account and models its stochastic nature. This is essential for our aim to capture the risk arising from deviation in the demand curve.

Second, the calibration of the model results in a very good fit on the calibration data, as well as on other sample data. This good empirical fit is very important to receive realistic results in our applications. Thus, the SMaPS model is well suited for an applied work.

6 A Model for an Energy-RAROC

Now we want to develop a model to calculate the RAROC of an electricity contract using a Monte Carlo Simulation based approach. In the first section we discuss the appropriate risk measure for our situation. In the second section we formulate the RAROC equation assuming a deterministic customer load. This assumption is relaxed in the third section and we show how we can model the systematic risk of one individual customer to get stochastic load paths. In the last section, we will show how part of the risk can be hedged, using the EEX future market and how we can reduce our exposure to risk by doing so.

6.1 The appropriate Risk Measure: VaR or CFaR?

As described, in banking business the Economic Capital is usually calculated as VaR - the Value at Risk. However, in our case VaR is not an appropriate measure of risk because, when using VaR, it is implicitly assumed that it is possible to close the risky position at any time on the future or forward market. In the energy business this cannot be done, because the market is not even close to liquid. Hourly products can only be traded on the spot market (or OTC) and even monthly contracts go only half a year ahead. Furthermore the amount of energy traded on the future market is also very limited.

Thus, we will use a similar, but slightly different measure - the Cash Flow at Risk, CFaR[12]. The difference to VaR is that we do not assume it is possible to close ones position at any time, but we have to wait until the maturity day is reached. If we do not have an own electricity production, we have to buy the electricity at the spot market. Here the difference between VaR and CFaR becomes clear: VaR is based on the future prices, CFaR on the spot prices. Since there is no future market for products with a granularity of hours and the OTC market for those products is not liquid, the usage of CFaR makes more sense when dealing with those products, which will be our main topic. Of course, VaR also has its applications in the electricity business, e.g. when dealing with monthly or yearly contracts.

We do not want to conceal at this point that VaR and thus also CFaR, have several drawbacks. The most important one is that the VaR (CFaR) is blind for the developments in the tail. Everything happening beyond the quantile is neglected. We discuss this issue in chapter 10 and also suggest another risk measure, the Expected Tail Loss (ETL), which do not suffer from the same problems as VaR (CFaR). The reason why we use CFaR in our first approach is its wide acceptance and popularity not only in the business world, but also on the regulatory side.

6.2 RAROC with deterministic Load

We now want to start developing a model to calculate the RAROC for the electricity business with hourly granularity. In this section we assume a very

[12]In the literature it is also known as Earnings at Risk (EaR) or Profit at Risk (PaR).

simple setting: We are an electricity trader, i.e. neither do we have any own facilities to produce electricity nor do we have any usage for it.

Assume we have a customer who wants to buy electricity from us for a fixed amount of money per unit. Furthermore assume in the beginning that his demand load is fixed and known, i.e. deterministic (we will relax this assumption in the next section).

Remember the original RAROC equation:

$$RAROC = \frac{\text{Expected Return}}{\text{Economic Capital}} \tag{6.1}$$

In chapter 2 we described how this equation can be used considering credit risk. Now the situation is different, thus, we want to explain how the numerator and the denominator of this fraction can be determined.

First we will determine the numerator. The Expected Return can be calculated as the expected value of the cash flows in the future. Say, we agreed to deliver energy for one year to our customer for a fixed retail price K, his (deterministic) load curve is $\hat{l}_t$ and the (stochastic) future spot price of one MWh at time t is S_t. Then the profit[13] of each hour is the difference between the retail and the spot price per MWh times the amount of energy. This is the future cash-flow in hour t CF_t. Since S_t is stochastic, CF_t is also stochastic.

$$\mathbb{E}[CF_t] = \mathbb{E}[(K - S_t)\hat{l}_t] = Kl_t - \mathbb{E}[S_t]\hat{l}_t \tag{6.2}$$

To get the entire profit we just have to sum over all hours from the starting date τ of the contract until the end date T and discount the cash flows to the actual point in time, which we denote with t_0. For simplicity we assume a constant interest rate r with continuous compounding[14].

$$\begin{aligned} \mathbb{E}[Profit] &= \mathbb{E}\left[\sum_{t=\tau}^{T} e^{-r(t-t_0)} CF_t\right] \\ &= \sum_{t=\tau}^{T} e^{-r(t-t_0)} \mathbb{E}\left[(K - S_t)\hat{l}_t\right] \end{aligned} \tag{6.3}$$

For each price path we can now calculate the profit (i.e. the sum of all cash flows). Thus, our best estimate for the expected profit is the mean of all profit realizations.

[13]Note that when talking about profit, we also include negative profits, i.e. losses.

[14]This assumption is made because the impact of the interest rate is not the core point of our analysis. Using a non-constant interest rate model would rise the problem of even more parameters to calibrate (we already have a three factor model with 10 parameters). The hourly compounding is also a simplification since payments are not done hourly in the real business world. Contracts at the EEX are settled daily, direct retail contracts with customers are usually settled monthly. Since payments dates differ among customers one would have to evaluate each contract differently.

As described in chapter 2, the Economic Capital should be the amount of money we can lose in a worst case scenario. We want to ensure that even under a very bad development we will still have enough capital to ensure the survival of the company. Therefore the Economic Capital should be invested in a risk free asset, e.g. German government bonds, to cover unexpected losses. We explained in section 5.1 that $CFaR_\alpha$ rather than VaR_α should be used to determine the Economic Capital for a project with hourly granularity. We decided to use the so-called relative $CFaR_\alpha$ (see (Dow98))), which is defined as the difference between the mean and the α-quantile of the profit and loss distribution. Thus, the RAROC of an energy project becomes:

$$\begin{aligned} RAROC &= \frac{ExpectedProfit}{CFaR_\alpha} \\ &= \frac{\mathbb{E}[Profit]}{\mathbb{E}[Profit] - q_\alpha[Profit]} \\ &= \frac{\sum\limits_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}[(K-S_t)\hat{l}_t]}{\sum\limits_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}[(K-S_t)\hat{l}_t] - q_\alpha[\sum\limits_{t=\tau}^{T} e^{-r(t-t_0)}(K-S_t)\hat{l}_t]} \end{aligned} \tag{6.4}$$

where q_α denotes the α-quantile.

6.3 RAROC with stochastic load

6.3.1 Systematic vs. Unsystematic Risk

The load process of a full load contract customer is generally not, as assumed before, deterministic. We do not know the future load process, however we are able to estimate the load curve with the help of historical data. This estimation can be used to compute an ex-ante RAROC as described in the previous section. But how can we model the uncertainty of the load process?

Deviations from the estimated load curve $\hat{l}_t$ can have various reasons. Similar to the concepts known from modern capital market theory, we want to distinguish between systematic and unsystematic reasons.

- Unsystematic reasons are caused by specific incidents at the customer and do not have their source in the market (e.g. a malfunction of a big machine, short-term variation in production activities, etc.).
- Systematic reasons, on the other hand, originate from variation in the market which have an impact on all customers (e.g. a cold snap).

Written as formula this means for the load l_i of customer i:

$$l_i = \hat{l}_i + \beta_i \varepsilon_{syst} + \varepsilon_i \tag{6.5}$$

where ε_{syst} is the systematic risk of the market, β_i describing the intensity of correlation between the customer an the systematic risk and ε_i is the unsystematic risk of customer i. Note that by definition the unsystematic risk is only related to the customer himself, there is no connection to other customers, i.e. for any two customers i and j:

$$Cov(\varepsilon_i, \varepsilon_j) = 0 \tag{6.6}$$

Furthermore, the unsystematic risk of each customer i is uncorrelated with the systematic risk, i.e.:

$$Cov(\varepsilon_i, \varepsilon_{syst}) = 0 \tag{6.7}$$

An electricity trader with a big portfolio of customers can be regarded as well diversified. That means the risk of variations due to unsystematic reasons (unsystematic risk) of all customers together can be assumed to compensate each other in average.

Hence, the only risk factor is the variation due to systematic reasons (systematic risk), which can be explained by variation in the entire grid load.

6.3.2 Modelling stochastic load paths

In this section we describe how we model stochastic load curves for each individual customer. As described before the spot market simulation model SMaPS developed by (BKMS04) is based on three stochastic processes. One of them is the grid load process L_t which is based on the apporach

$$L_t = \hat{L}_t + L'_t \tag{6.8}$$

where $\hat{L}_t$ is the deterministic grid load forecast for Germany and L'_t is a SARIMA time series model with 24h seasonality. Figure 3 shows three exemplary load paths.

To use this process for generating simulations for the customer's load process we have to estimate the customer's correlation with the entire grid load first.

To do so, we determine the impact of fluctuations of the grid load on the customer load, precisely the portion of deviation of the customer load from the estimated load which can be explained by the deviation of the German grid load from the estimated grid load, i.e. we conduct a simple linear regression. Set

$$\tilde{l}_t = \frac{l_t - \hat{l}_t}{\hat{l}_t} \tag{6.9}$$

and

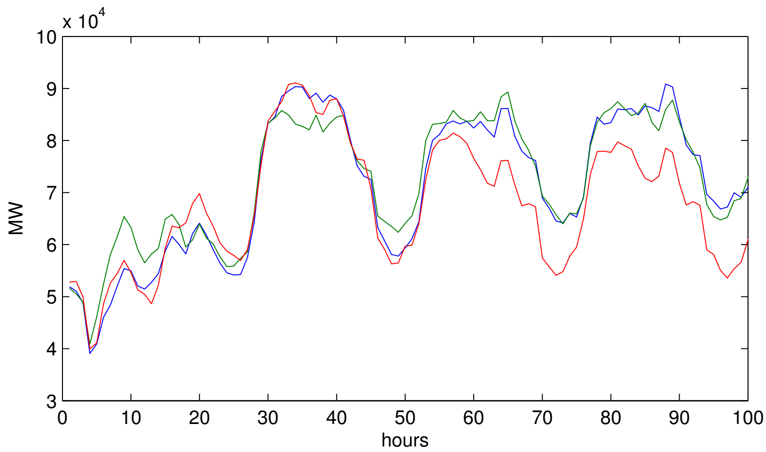

Figure 3: Cutout of three arbitrary load paths generated with SMaPS

$$\tilde{L}_t = \frac{L_t - \hat{L}_t}{\hat{L}_t} \tag{6.10}$$

then the regression model can be written as:

$$\tilde{l}_t = \beta \cdot \tilde{L}_t + \varepsilon_t \tag{6.11}$$

where
l_t = actual customer load
$\hat{l}_t$ = estimated customer load
L_t = actual entire grid load
$\hat{L}_t$ = estimated entire grid load
β = regression coefficient
ε_t = error term (unsystematic risk)

The value of β can be computed by taking the covariance of $\tilde{l}_t$ and $\tilde{L}_t$[15]:

$$\begin{aligned} Cov(\tilde{l}_t, \tilde{L}_t) &= Cov(\beta \tilde{L}_t + \varepsilon_t, \tilde{L}_t)) \\ &= Cov(\beta \tilde{L}_t, \tilde{L}_t) + Cov(\varepsilon_t, \tilde{L}_t) \\ &= \beta \cdot Var(\tilde{L}_t) + 0 \end{aligned} \tag{6.12}$$

Thus we get for β:

[15]By definition $\tilde{L}_t$ is independent of ε_t

$$\beta = \frac{Cov(\tilde{l}_t, \tilde{L}_t)}{Var(\tilde{L}_t)} = \varrho_{\tilde{l}_t, \tilde{L}_t} \frac{\sigma_{\tilde{l}_t}}{\sigma_{\tilde{L}_t}} \tag{6.13}$$

where $\varrho_{\tilde{l}_t, \tilde{L}_t}$ denotes the correlation coefficient and σ the standard deviation of $\tilde{l}_t$ and $\tilde{L}_t$, respectivly.

To compute a customer's beta we need $\tilde{L}_t$ and $\tilde{l}_t$, the deviations of grid and customer load. To get them we will use historical data to make a load estimation for a year, for which we also have the realized load curve available. To do this we classify each day according to the following scheme:

- Monday
- Tuesday, Wednesday or Thursday
- Friday
- Saturday
- Sunday

Tuesday, Wednesday and Thursday are put together into one group since the load curves of them are historically very similar. We do this classification for each month and additionally we distinguish holidays as Easter, 1st of May, 3rd of October, Christmas holidays, etc. Doing this we get 82 classes of days. We break this scheme down for each hour, so in the end we have 1968 classes.

Our best load estimation for the future load is the average of the values in the same class in former years. To allow an easy computation we use the regression function of a statistical software and the following model equation:

$$\begin{pmatrix} l_1 \\ \vdots \\ l_t \\ \vdots \\ l_T \end{pmatrix} = \begin{pmatrix} d_1(1) & \cdots & d_k(1) & \cdots & d_K(1) \\ \vdots & & \vdots & & \vdots \\ d_1(t) & \cdots & d_k(t) & \cdots & d_K(t) \\ \vdots & & \vdots & & \vdots \\ d_1(T) & \cdots & d_k(T) & \cdots & d_K(T) \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_k \\ \vdots \\ \alpha_K \end{pmatrix} \tag{6.14}$$

with

$$d_k(t) = \begin{cases} 1 \text{ , hour } t \text{ is in class } k \\ 0 \text{ , else} \end{cases} \tag{6.15}$$

l is a T-dimensional vector with the hourly load curve of the past and α a K-dimensional vector which will contain the average values for each class of hour. Having this, we can calculate the beta of each customer using (6.13).
Equipped with a beta for each customer we can generate stochastic load paths depending on the systematic risk of each individual customer. For this we generate grid load paths according to (6.8) and compute the relative deviation λ_t

from the mean $\hat{L}_t$ for each path L_t^i, i.e.:

$$\lambda_t^i = \frac{L_t^i - \bar{L}_t}{\bar{L}_t} \tag{6.16}$$

Having done this for each hour and each path we can now generate i different load paths for the customer load by multiplying the estimated load path $\hat{l}_t$ with $\beta\lambda_t^i$ and adding this deviation to the estimated load $\hat{l}_t$:

$$l_t^i = \hat{l}_t + \hat{l}_t \cdot \beta \cdot \lambda_t^i \tag{6.17}$$

The profit function is now depending on two sources of uncertainty: The spot prices and the customer load curve. The expected value is given by:

$$\begin{aligned}
\mathbb{E}[Profit] &= \sum_{t=\tau}^{T} e^{-r(t-t_0)} \mathbb{E}\left[CF_t\right] \\
&= \sum_{t=\tau}^{T} e^{-r(t-t_0)} \mathbb{E}\left[(K - S_t) l_t\right] \\
&= \sum_{t=\tau}^{T} e^{-r(t-t_0)} \left(K\mathbb{E}\left[l_t\right] - \mathbb{E}\left[S_t\right]\mathbb{E}\left[l_t\right] - Cov(S_t, l_t)\right) \quad (6.18)
\end{aligned}$$

Here, we see that in order to evaluate the expected value of the profit, we even do not need to generate simulations for the customer load. Having the simulated grid load paths is sufficient, since $\mathbb{E}[l_t] = \hat{l}_t$ and using (6.11), (6.18) becomes:

$$\begin{aligned}
\mathbb{E}[Profit] &= K\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}[S_t]\hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)} Cov[l_t, S_t] \\
&= K\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}[S_t]\hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t Cov(\tilde{l}_t, S_t) \\
&= K\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}[S_t]\hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t \beta Cov(\tilde{L}_t, S_t)
\end{aligned} \tag{6.19}$$

This result is very helpful when we want to compute the Covariance between the spot price S_t and a customer load l_t. We can do this by multiplying the Covariance of spot price and grid load with the customer's beta and the load estimate $\hat{l}_t$. We will use this result in the next chapter.

Unfortunately if we want to compute the deal's RAROC we still need to generate load paths for the customer because not only the mean but also the α-quantile enters the calculation.

6.4 RAROC with the possibility of Hedging

6.4.1 Energetic Hedging

In the previous section we assumed that there is no future market and thus no possibility to hedge the risk. This, however, is not true in reality. There is a market for future contracts (in Germany the EEX and various brokers, see chapter 3), but the justification to use CFaR and not VaR as a measure of risk still holds. As described before, there are only monthly, quarterly and yearly future contracts available, nevertheless we want to calculate RAROC on an hourly basis. Thus, it is only possible to hedge some of the risk but not all of it. Two different products can be used for hedging: Baseload and Peakload future contracts. Remember that a Baseload contract means the constant delivery of 1 MW 24h hours a day, seven days a week. A Peakload contract includes the delivery of 1 MW from 8:00am to 8:00pm Monday through Friday (including holidays).

But which hedging strategy should we follow? One intuitive solution (and physically meaningful) to this problem is to follow a so-called energetic hedge strategy. This means we buy a future on the same amount of total energy we are going to sell to our customer. When dealing with stochastic load paths, we take the average values to compute the sum of energy.

Let $\eta = (\eta_{peak}, \eta_{base})$ denote the energetic hedge strategy where η_{peak} and η_{base} denote the number of Peakload and Basleload contracts bought or sold, respectively. This strategy can be calculated as following:

$$\eta_{peak} = \frac{\sum_{t=\tau}^{T} l_t 1_{\{t \in \text{peak}\}}}{(T-\tau)1_{\{t \in \text{peak}\}}} \tag{6.20}$$

and

$$\eta_{base} = \frac{\sum_{t=\tau}^{T} l_t - \sum_{t=\tau}^{T} l_t 1_{\{t \in \text{peak}\}}}{(T-\tau) - (T-\tau)1_{\{t \in \text{peak}\}}} \tag{6.21}$$

where $1_{\{t \in \text{peak}\}}$ denotes the indicator function, i.e.:

$$1_{\{t \in \text{peak}\}} = \begin{cases} 1 \text{ , if t is a Peakhour} \\ 0 \text{ , else} \end{cases} \tag{6.22}$$

Figure 4 shows an exemplary loadcurve and the energetic hedge position for it. As we are short in the load and long in the hedge position, only the difference remains as risky position. This new load curve is shown in figure 5. A negative load means that we are going to sell the energy at the exchange. If π_{peak} and π_{base} denote the prices of Peakload and Baseload future contracts, the profit and loss function becomes:

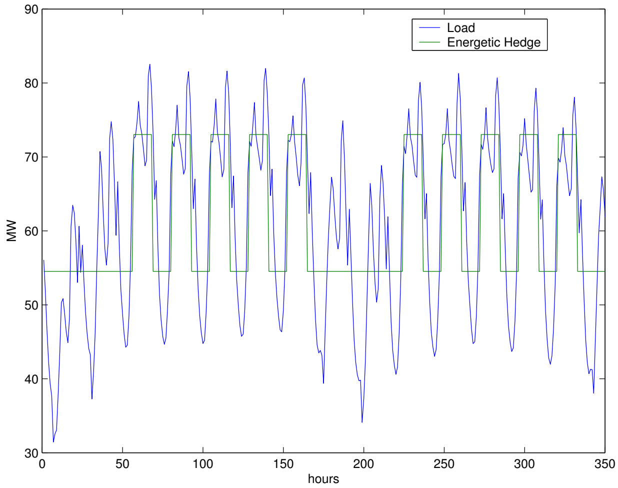

Figure 4: Energetic Hedge for a typical customer load

$$
\begin{aligned}
Profit = \quad & K\sum_{t=\tau}^{T} e^{-r(t-t_0)} l_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)}(\eta_{base}\pi_{base} + \eta_{peak}\pi_{peak}1_{\{t\in\text{peak}\}}) \\
& + \sum_{t=\tau}^{T}(e^{-r(t-t_0)}((\eta_{base} + \eta_{peak}1_{\{t\in\text{peak}\}}) - l_t)S_t) \qquad (6.23)
\end{aligned}
$$

where everything is known at time τ except of the price process S_t and the load process l_t. Plugging (6.23) into (6.4) we can compute the new RAROC.

We assume that the future prices given by the market are fair, i.e. they reflect the average future spot prices. (We achieve this by adjusting the spot price simulations by the actual future prices). If we also assume that there are no transaction costs, especially no bid-ask spread, then we know that the expected value of the profit given by (6.23) will not change with the hedging strategy. Otherwise there would be an opportunity for arbitrage in the market.

Thus, the expected value will not change but the quantile of the distribution will. The distribution will become denser and the quantile will lie much closer to the mean. Unfortunately the liquidity of the future market is very limited what makes hedging for big positions in reality often difficult.

6.4.2 Determining the optimal Hedging Strategy

As described before, the energetic hedge is the best strategy from an engineering point of view. But since the price is not constant this does not have to be the optimal strategy in the economic sense. If the maximization of the RAROC is

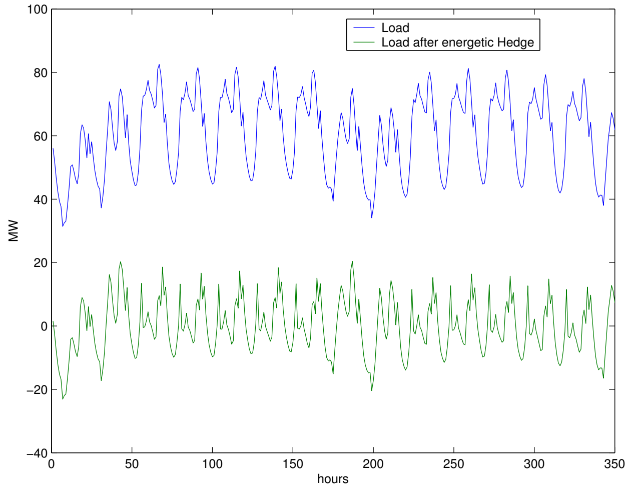

Figure 5: Load after entering an energetic hedge position

our objective the optimization problem can be written as:

$$\max_{\theta} F = \frac{\mathbb{E}[Profit]}{\mathbb{E}[Profit] - q_\alpha[Profit]} \tag{6.24}$$

where θ stands for the hedging strategy $(\theta_{peak}, \theta_{base})$. θ_{peak} and θ_{base} are the number of Peakload and Baseload future contracts to be bought or sold.

This problem cannot be solved with a closed formula but with the Monte Carlo Simulation based approach described in the last section. We implemented the optimization problem in MATLAB. Given the load curve shown in figure 6 the energetic hedge strategy is $\eta = (19.00, 54.03)$ (using 6.20 and 6.21) and the optimal hedge strategy is $\theta = (17.19, 57.40)$. Figure 6 shows an energetic and an optimal hedge for a given customer load.

Having developed a model for an Energy-RAROC and also determined possible hedging strategies we want to continue in the next chapter with deriving formulas for risk premiums for full load electricity contracts.

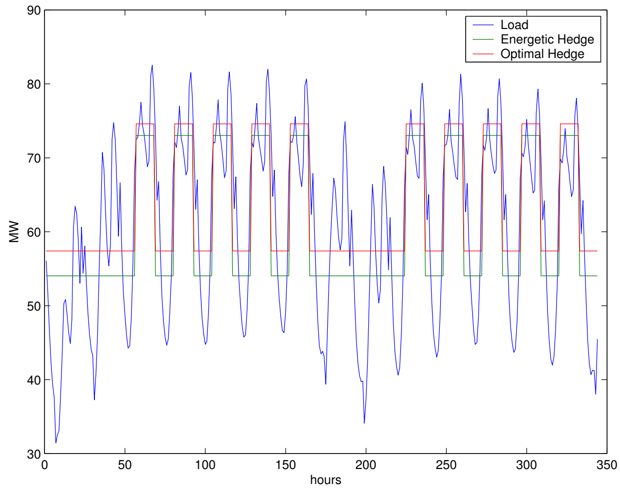

Figure 6: Optimal Hedge for a typical customer load

7 Risk Premiums of Full Load Contracts

We have stated in the beginning that our intention is to evaluate full load contracts. Closing such a deal the supplier accepts to take over several types of risks. As compensation he demands premiums in addition to the basic price. The overall price is given by the sum of the basic price and three risk premiums, thus we have four components:

- The basic price
- A risk premium for the hourly spot market price risk
- A risk premium for the volume risk
- A risk premium due to the price-volume correlation

Our focus are the risk premiums and not the price itself. In the following section we will explain what types of risks are covered with each of the premiums. We will then show how the framework developed in the previous section can be used to compute these premiums.

7.1 Market Price Risk

The market price risk has its source in the volatile spot market. When entering a delivery contract, we do not know the future spot prices, but we decide about the retail price on the signing day. That means we accept to bear the risk of hourly changing market prices on behalf of the customer. Part of this risk we can hedge by buying opposite future contracts at the EEX. But since only Baseload and Peakload contracts for months, quarters and years are available and the customer load curve changes hourly one can only hedge part of the risk. For the remaining risk we are accepting to bear for the customer we want to get paid a risk premium. How much should this premium be?

To determine the market price risk premium, we use the same setting as in the first part of the previous section: A deterministic load curve $\hat{l}_t$ and a stochastic spot price process S_t. Here the price process is the only source of uncertainty. We will calculate the risk premium as the difference between a fair retail price regarding the risky nature of the contract and the fair retail price neglecting this risk.

The "fair" retail price K per MWh without considering the market price risk is the price of K such that the expected value of the P&L function becomes zero. We denote this "fair" price with K_1. We use risk neutral valuation to price the contract. It can be computed using (6.3):

$$\mathbb{E}^*[Profit] = 0 \Leftrightarrow K_1 = \frac{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t\mathbb{E}^*[S_t]}{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t} \tag{7.1}$$

On the other hand, what price should we take if we take the market price risk into consideration. As stated before, a project is valuable for us, i.e. adds economic value, if its RAROC is higher than an internal hurdle rate. A RAROC below the hurdle rate would destroy economic value and thus would not be desirable for the company. Using this RAROC-based approach we can calculate a retail price K_2 which results in a RAROC equal to our hurdle rate. If μ denotes the internal hurdle rate, we compute K_2 using the condition:

$$RAROC = \mu \tag{7.2}$$

Plugging in (6.4) we get:

$$\frac{K_2 \sum_{t=\tau}^{T} e^{-r(t-t_0)} \hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)} \mathbb{E}^*[S_t] \hat{l}_t}{K_2 \sum_{t=\tau}^{T} e^{-r(t-t_0)} \hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)} \mathbb{E}^*[S_t] \hat{l}_t - q^*_\alpha [K_2 \sum_{t=\tau}^{T} e^{-r(t-t_0)} \hat{l}_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)} S_t \hat{l}_t]} = \mu \tag{7.3}$$

Solving for K_2 this leads us to:

$$K_2 = \frac{\mu \left(q^*_{1-\alpha} \left[\sum_{t=\tau}^{T} e^{-r(t-t_0)} S_t \hat{l}_t \right] - \sum_{t=\tau}^{T} e^{-r(t-t_0)} \hat{l}_t \mathbb{E}^*[S_t] \right) + \sum_{t=\tau}^{T} e^{-r(t-t_0)} \hat{l}_t \mathbb{E}^* \left[S_t\right]}{\sum_{t=\tau}^{T} e^{-r(t-t_0)} \hat{l}_t} \tag{7.4}$$

The value K_2 gives us the fair price if we require the internal hurdle rate μ. Thus we can determine the premium we want to receive per MWh due to our exposure to spot market price risk p_m as:

$$\begin{aligned} p_m &= K_2 - K_1 \\ &= \frac{\mu \left(q^*_{1-\alpha} \left[\sum_{t=\tau}^{T} e^{-r(t-t_0)} S_t \hat{l}_t \right] - \sum_{t=\tau}^{T} e^{-r(t-t_0)} \hat{l}_t \mathbb{E}^*[S_t] \right)}{\sum_{t=\tau}^{T} e^{-r(t-t_0)} \hat{l}_t} \end{aligned} \tag{7.5}$$

This is exactly the same as the Economic Capital multiplied by μ and divided by the total amount of energy what makes perfect sense: We demand as premium the return of μ on the capital we need to put aside due to the risky nature of the deal. Dividing by the total amount of energy just standardize the total premium for the contract to the premium per MWh so that we are able to compare contracts with different amounts of energy.

7.2 Volume Risk

When entering a full load contract one do not only take over market price risk, but also volume risk, since one allows the customer to use as much energy as he wants. To determine the premium for this risk, we follow the same approach as in the previous section, but we use stochastic load curves. The price K_3 is the price leading to a zero expected profit, i.e.:

$$\mathbb{E}^*[Profit] = 0 \Leftrightarrow K_3 = \frac{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*[S_t l_t]}{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*[l_t]} \tag{7.6}$$

K_3 is the fair price disregarding market price and volume risk. If we take them into consideration, we can determine a price K_4, which leads to a RAROC equal to the hurdle rate, i.e. we require:

$$\frac{K_4 \sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*\left[l_t\right] - \sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*\left[S_t l_t\right]}{K_4 \sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*\left[l_t\right] - \sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*\left[S_t l_t\right] - q_\alpha^* \left[K_4 \sum_{t=\tau}^{T} e^{-r(t-t_0)} l_t - \sum_{t=\tau}^{T} e^{-r(t-t_0)} S_t l_t\right]} = \mu \tag{7.7}$$

Unfortunately, this equation cannot be solved analytically for K_4, so we have to use numerical methods to compute a value for K_4. Having done this using numerical procedures of MATLAB, we can determine the risk premium for the volume risk. The difference between K_4 and K_3 captures both, the volume as well as the market price risk. Subtracting the market risk premium we get the volume risk premium p_v:

$$p_v = K_4 - K_3 - p_m \tag{7.8}$$

7.3 Price-Volume Correlation Risk

The last component is the risk premium for the correlation of price and volume of the customer demand. Typical customers tend to have an increasing demand at times when prices are high. The reason for this is clear: Both processes are driven by the same underlying factor.

Of course also the opposite is possible. A customer could control his demand load such that it is lower than the average when the overall grid load is higher. Given that he as a full load contract with a fixed price, this seems very unlikely.

The risk due to price-volume correlation can be calculated by comparing K_3 and K_1. If we evaluate the expected value in equation (7.6) we get:

$$K_3 = \frac{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*[S_t]\mathbb{E}^*[l_t] + \sum_{t=\tau}^{T} e^{-r(t-t_0)}Cov^*(l_t, S_t)}{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*[l_t]} \tag{7.9}$$

Subtracting (7.1) we get the change in price due to the systematic correlation between the load l_t and the spot price S_t. This is the risk premium p_c:

$$\begin{aligned} p_c &= K_3 - K_1 \\ &= \frac{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*[S_t]\mathbb{E}^*[l_t] + \sum_{t=\tau}^{T} e^{-r(t-t_0)}Cov^*(l_t, S_t)}{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\mathbb{E}^*[l_t]} - \frac{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t\mathbb{E}^*[S_t]}{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t} \\ &= \frac{\sum_{t=\tau}^{T} e^{-r(t-t_0)}Cov^*(l_t, S_t)}{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t} \end{aligned} \tag{7.10}$$

Using the same substitution as in 6.19 we can write:

$$p_c = \frac{\beta \sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t Cov^*(\tilde{L}_t, S_t)}{\sum_{t=\tau}^{T} e^{-r(t-t_0)}\hat{l}_t} \tag{7.11}$$

Note that the premium p_c is different to the premiums p_m and p_v. To compute p_c we do not need the RAROC approach as risk measure like for the other two premiums. We just compare two average values using the deterministic and the stochastic load curves. Neither do we need the internal hurdle rate μ nor the quantile of the profit and loss distribution. Thus even if we decide to calculate our premiums in a different framework, (7.10) will stay the same.

7.4 Overview over the Risk Premiums

In the previous sections we have shown how the three risk premiums for market price risk, volume risk and price-volume correlation risk can be computed. We want to summarize the results in the following overview. The entire risk premium p_R is given by:

$$p_R = p_m + p_v + p_c \tag{7.12}$$

and the single premiums can be computed as:

$$\begin{aligned} p_m &= K_2 - K_1 && (7.13) \\ p_v &= K_4 - K_3 - K_2 + K_1 && (7.14) \\ p_c &= K_3 - K_1 && (7.15) \end{aligned}$$

hence

$$p_R = K_4 - K_1 \qquad (7.16)$$

The premium p_R is the amount of money per load unit we should charge our customer additionally to our production costs and profit margins to compensate us for the risk we have taken with the obligation to deliver as much energy as the customer wants for a fixed price.

Note that there are also other kinds of risks involved which we did not consider. These are, for example, model risk, operational risk and risk related to reserve energy.

8 Model Improvement: ETL

8.1 Drawbacks of VaR and CFaR

Although VaR and CFaR have become popular in the financial industry because they are easy to employ and provide a single number to measure the entire risk, the drawbacks of using VaR (CFaR) are often overseen. There are many problems when using VaR (CFaR) as risk measure:

First, the usage of VaR (CFaR) neglects what is happening behind the α-quantile in the tail of the distribution. If the distribution is heavy-tailed (leptocurtic) this can lead to severe underestimations of the exposure to risk. When decreasing the VaR (CFaR) it is possible that the mean beyond the quantile, i.e. the expected loss, given a loss bigger than VaR is increasing.

Another problem dealing with VaR (CFaR) is that it is not subadditive, i.e.

$$VaR(X + Y) \leq VaR(X) + VaR(Y) \tag{8.1}$$

is not true in general. From an economic point of view this does not make sense. It means that it is possible that the sum of two single portfolios has a lower risk than the two portfolios together. To see how this can happen consider the following example: We are holding two independent securities A and B with the same maturity date. At the maturity, A pays 100 Euro with 96% probability and 0 Euro with 4% probability. Security B pays 100 Euro with 97% probability and 0 Euro with 3%. For both securities the VaR at the 5% level is 0, i.e. $VaR(A) = 0$ and $VaR(B) = 0$. On the other hand the VaR of both securities together is higher than zero, since the probability that both securities do pay 100 Euro back is only 93.12%.

Third, optimizing a portfolio with respect to its VaR (CFaR) is a non-convex optimization problem. The optimization function has multiple local minima why one would have to use time-consuming optimization algorithms to find the global optimum.

Especially when dealing with heavy-tailed distributions the drawbacks of VaR become significant. Thus we suggest another risk measure in the next section, known as Expected Tail Loss, short ETL.

8.2 Expected Tail Loss (ETL)

The Expected Tail Loss (ETL), also known as Conditional VaR (CVaR) or Expected Shortfall (ES), was invented to correct the above mentioned drawbacks of VaR. It is defined as the expected value of the tail, i.e. the distribution beyond VaR. In contrast to VaR, it fulfills the subadditivity. More important, it is not blind for the "events" in the tail of the profit and loss distribution. If the distribution of the tail changes, the ETL also changes in the correct way (i.e. if higher losses in the tail become more likely, ETL increases and vice versa).

	VaR	ETL
Sub-Additivity	No	Yes
Convex Portfolio Optimization	No	Yes
Blind in the Tail	Yes	No

Table 1: Comparison of VaR and ETL

Formally ETL is defined as:

$$ETL(Profit)_\alpha = \mathbb{E}[-Profit| - Profit > VaR_\alpha(Profit)] \tag{8.2}$$

ETL is not only subadditive but also fulfills the other conditions of a coherent risk measure as defined by (ADEH99). The coherence of ETL (CVaR, ES) was proven by (Pfl00). Besides the coherence (RU00) show that the ETL is, in contrast to the VaR, a smooth, convex function what allows them to use linear programming technique to optimize a portfolio. Thus, using ETL instead of VaR (CFaR) enables us two improve our model in three ways: First, the optimization problem to find the optimal hedging strategy can be solved unambiguously. Second, a portfolio approach can be taken without facing the problem of non-subadditivity of the VaR (CFaR). Finally, and perhaps most important, the risk management will be improved since one do not neglect the information given by the tail of the profit and loss distribution. Table 1 summarizes the superiority of ETL over VaR.

8.3 The improved Model

Having discussed the disadvantages of VaR (CFaR) compared to ETL, we want to improve our model. To do so we will use ETL as the new measure of risk. As before we will use the cash-flow-based value rather than the future-based one. That means we have to substitute VaR_α in (8.3) by $CFaR_\alpha$. We denote this new quantity with $CFETL_\alpha$

$$CFETL(Profit)_\alpha = \mathbb{E}\left[-Profit| - Profit > CFaR_\alpha(Profit)\right] \tag{8.3}$$

The RAROC based on the $CFETL$ rather than on the $CFaR$ becomes:

$$RAROC = \frac{\mathbb{E}[Profit]}{CFETL_\alpha[Profit] - \mathbb{E}[Profit]} \tag{8.4}$$

Basically we can keep our formulas the same, we just have to substitute $CFaR_\alpha$ by $CFETL_\alpha$. Having done this substitution in our MATLAB algorithms we can compute the new risk premiums based on $CFETL_\alpha$.

9 Conclusion and Final Remarks

The aim of this thesis has been to use the RAROC framework developed in the banking business to evaluate electricity contracts (mainly full load contracts) and to calculate risk premiums the supplier should charge the customers as compensation for the risks related to them.

We showed how a Monte Carlo Simulation based approach can be taken to calculate premiums related to the risks of hourly changing market prices, changing load volumes and price-volume correlation. The market price risk premium has been calculated as difference between two selling prices. First the "fair" price, resulting in an expected payoff of zero, second the price resulting in a RAROC equal to an internal hurdle rate. Fort the volume risk and the price-volume correlation we needed load simulations for each customer. To get them, we estimated the systematic risk of a customer by computing the correlation between the customer's load and the grid load. Having the correlation between the two load processes and simulation paths for the grid load, we were able to generate simulation paths for the customer's load, reflecting only the systematic risk. Unsystematic risk was assumed to be diversified, since an electricity supplier has a big portfolio of many customers.

We explained, why the risk measure we used so far is not the best one, even if it is well-established in the industry. We proposed to use another risk measure, the Expected Tail Loss and explained why it is superior to the Value at Risk (Cash Flow at Risk).

In the last section, we showed that also other spot price models can be used. Price models based on the α-stable distribution provide a good fit not only in financial but also in electricity price modelling. Unfortunately the α-stable model did not provide load simulations so only the market price premium could be calculated. Further work has to be done on this field, e.g. by modelling the error terms of the SARIMA models of SMaPS with the α-stable instead of the gaussian distribution.

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