Generalized Asset Value Credit Risk Models and Risk Minimality of the Classical Approach

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Abstract:

We place the asset value credit portfolio model in the larger context of generalized correlation models where the normal distribution assumption of asset returns is replaced by an abstract elliptical distribution.

Based on closed-form solutions for homogenous portfolios, we show in particular that the classical asset value model is not robust against misspecifications of the assumed asset return distribution, that it further systematically underestimates portfolio risk, if the asset return distribution is non-normal, and that it may also induce insufficient supply of economic capital to cover credit portfolio risk in the world's financial institutions.

Keywords: credit risk, credit portfolio modeling, credit portfolio risk, asset value model, elliptical distribution, asymptotic loss distribution, loss density, value at risk, model error, economic capital

JEL Classification: C13, C15, C16, C51, G33

1. Introduction

A crucial step in the construction of a credit portfolio model is the description of dependencies between clients. One of the most widespread credit portfolio risk models in the banking industry world wide is the so called asset value model which goes back to an article by Robert Merton of 1974[1] and was then further developed by Oldrich Vasicek and Stephen Kealhofer of KMV Corporation[2] in the mid 1990's and by Mickey Bhatia, Christopher Finger and Greg Gupton under the name of Credit Metrics in 1997[3].

The asset value model is mainly a reinterpretation of the classical Black-Scholes-Merton option pricing model in a credit risk context. For this reason, the assumption of a geometric Brownian motion as the mathematical model for asset price processes and the normality of marginal asset returns play a central role in the asset value model.

In this article, we will place the classical asset value model, to which we shall also refer as the normal correlation model, in the larger context of generalized correlation models where the normal distribution assumption of asset returns is replaced by an abstract elliptical distribution.

As the normal distribution, multivariate elliptical distributions are in general entirely defined by the type of their marginal distributions and their linear covariance matrix and in that respect are a straightforward generalization of the normal correlation model.

Extending a result of Vasicek (1991), we use the closed-form solutions for the portfolio loss distributions of homogenous portfolios to show that the distributional assumption in the generalized correlation model, which contains the classical asset value model as a special case, has a significant influence on the reported risk of a given and unchanged credit portfolio. In particular, we show that it is the normal correlation model which detects the least risk of all versions of the generalized correlation model in a homogenous portfolio if the value at risk at a sufficiently high confidence level is used as portfolio risk measure. This implies that the classical asset value model is not robust against misspecifications of the assumed asset return distribution, that it further systematically underestimates portfolio risk, if the asset return distribution is non-normal, and that it may also induce insufficient supply of economic capital to cover credit portfolio risk in the world's financial institutions.

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1 See Merton (1974).

2 See Kealhofer (1993).

3 Gupton et al. (1997).

As further results, we describe the univariate symmetric distributions which are the marginal distributions of multivariate elliptical distributions in arbitrary dimensions as variance mixtures of normals. We give asymptotic analytic solutions for the portfolio loss distribution and the portfolio loss density of large homogenous portfolios in the generalized correlation model and state some of their properties. Finally, we show that the risk minimality property of the normal correlation model is unrelated to the tail independence of the bivariate normal distribution.

The paper is organized as follows: After first defining the generalized correlation model we then develop the loss distribution and loss density of homogenous portfolios and show some of their properties. Finally, we prove the risk minimality of the normal correlation model within the family of generalized correlation models[4]. All proofs are given in the appendix.

2. The normal correlation model

The normal correlation model was the first fully worked out model of dependencies between clients in a credit portfolio. It was mainly developed by Oldrich Vasicek and Stephen Kealhofer of KMV Corporation[5] in the mid 1990's.

Based on Robert Merton's seminal article on corporate default risk of 1974[6], it is assumed that a company defaults if its firm value falls below the face amount of its debt at the time the debt is due[7] because its proprietors are better off if they hand the firm over to the creditors instead of repaying the debt.

Moreover, in the Merton-model, the firm value follows a geometric Brownian motion. This supposition allows for a straight forward generalization of Merton's single firm model to a portfolio model by assuming the joint firm value processes of a set of n firms to follow a multivariate geometric Brownian motion with pairwise linear correlations ρ_{ij} between firm i and firm j, $i, j = 1,\dots, n$, of the logarithmic increments. Still every single firm defaults if its asset value is inferior to its liabilities at the debt's maturity; however, default events can now be dependent due to the correlated asset values.

4 Various software tools that illustrate the results are available for free at www.wehrspohn.de and www.risk-and-evaluation.com.

5 See Kealhofer (1993).

6 See also Wehrspohn (2002) sections I.B.1. and I.B.2.

7 It is also supposed that all of the firm's liabilities consist of zero bonds of the same seniority due at the same point in time. For a full list of assumptions of Merton's model confer to Wehrspohn (2002) section I.B.1.

As a simplification, KMV supposes that all firms' debts have a standardized time to maturity of one year. Since the logarithm of the firm value process, i.e. the asset return process, at time $t = 1$ is normally distributed with given mean and variance and since the linear correlation ρ is invariant under linear transformations[8], we can presume without loss of generality that the firms' joint asset return process V_1 is normally distributed with mean 0 and a covariance matrix that equals its correlation matrix.

Once default probabilities p_i, $i = 1,\ldots, n$, have been calculated for the single firms with the original Merton approach, the firms' debts can be replaced by abstract default thresholds d_i given by

$$d_i = \Phi^{-1}(p_i)$$

where $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of the standard normal distribution.

Now the firms' joint default behavior can be simulated by drawing random numbers from the multivariate standard normal distribution with the specified correlation matrix.

A variation of the KMV approach was suggested by Gupton, Finger and Bhatia in the model known as Credit Metrics[9]. In Credit Metrics the Vasicek-Kealhofer portfolio model is separated from Merton's option pricing method to calculate default probabilities. Default and also transition probabilities from one grade to the other are considered as being exogenously given through company ratings so that the firms' asset value processes are no longer relevant to fit the model. This enlarges the applicability of the model from public to all externally or internally rated companies.

The concept of dependence between counterparties in Credit Metrics is identical to the KMV approach. The interpretation of the multivariate normal and the marginal distributions as the firms' joint and marginal asset return distributions is now purely intuitive in the Credit Metrics context, so that it would be better to rather speak of abstract risk index distributions. Their main role in the model is to extend the individual firms' transition probabilities as given by their ratings to joint transition probabilities for the entire portfolio of firms.

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8 Note that this implies that asset return correlations are time invariant. The time horizon t only enters into the calculation of portfolio risk through the firms' default probabilities.

9 Gupton et al. (1997).

3. The generalized correlation model

While being economically intuitive, a major drawback of the normal correlation model is the somewhat arbitrary choice of the multivariate normal distribution to describe the joint movements of clients' individual risk indices. Historical reasons certainly were dominant in this selection because normal distributions appear as finite dimensional marginal distributions of the log-returns of the geometric Brownian motion, the standard model of continuous stochastic processes.

In the present discussion, however, a typical criticism of the normal distribution is that it is not well adapted to the specific features of a lot of financial data. This assessment refers especially to the phenomenon that many empirical distributions have long tails, i.e. that large deviations from the mean of a distribution are observed much more frequently than one would expect if the underlying distribution was normal.

In the normal correlation model two things were fundamental: the marginal distributions that were needed to calculate clients' default and transition thresholds and the correlation matrix of clients' risks indices. In order to extend the model, note that a multivariate distribution is in general not uniquely determined by its marginal distributions and its correlation structure. Exceptions in that respect, however, are spherical and elliptical distributions.

Definition 1:

A distribution D is called spherical[10] if it is invariant under orthogonal transformations, i.e. if for a random vector $X \in \mathbf{R}^n$ with $X \sim D$ and any orthogonal map $U \in \mathbf{R}^{n \times n}$ the equation

$$\mathrm{L}(X) = \mathrm{L}(UX)$$

holds[11]. If D has a density d, then this definition is equivalent to saying that d is constant on spheres.[12]

Definition 2:

Let $\boldsymbol{S}$ be the family of all spherical distributions. A distribution D is called elliptical[13] if it is an affine linear transformation of a spherical distribution[14], i.e. if for a random vector $X \in \mathbf{R}^n$

10 Or 'spherically symmetric'.

11 $\mathrm{L}(X)$ denotes the law of *X*. The expression denotes that the distributions of *X* and of *UX* are equal.

12 Fang et al. (1989), definition 2.1., p. 29.

with $X \sim D$ and a random vector $Y \in \mathbf{R}^n$ with $\mathrm{L}(Y) \in \boldsymbol{S}$ there exists $\mu \in \mathbf{R}^n$ and $A \in \mathbf{R}^{n \times n}$ so that

$$X = \mu + A \cdot Y .$$

The best known example of spherical or elliptical distributions respectively is the family of multivariate normal distributions so prominent in the normal correlation model.

Elliptical distributions are an interesting generalization of the normal distribution in the asset value model because a multivariate elliptical distribution is uniquely determined by its univariate marginals, its mean, and its covariance matrix since the type of all marginals is the same[15].

Not all symmetric univariate distributions are possible as marginal distributions of an elliptical distribution in $\mathbf{R}^n$ for any $n \in \mathbf{N}$. It can be shown, however, that a univariate distribution *D* is the marginal distribution of a spherical distribution in $\mathbf{R}^n$ for any $n \in \mathbf{N}$ if and only if it is a variance mixture of centered normals[16]. Hence[17], *D* can be defined by its density function

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{1}{\sqrt{s}} \exp\left(-\frac{x^2}{2s} \right) dW(s)$$

where the weight or mixing distribution *W* only takes values on $(0, \infty)$, i.e. a variance mixture of normals is a normal distribution with random variance. This definition immediately implies that a random variable $X \sim D$ can be written as

$$X = \sqrt{w} \cdot Y$$

where *Y* is standard normally distributed, $w \sim W$, and *Y* and *w* are stochastically independent.

Example 1

A well-known example of a normal variance mixture is the Student-t distribution with *n* degrees of freedom. Here the mixing distribution *w* is given as

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13 Or 'elliptically symmetric'.

14 Fang et al. (1989), definition 2.2., p. 31.

15 Cf. Embrechts et al. (1999), p. 11.

16 Fang et. al, (1989), theorem 2.21, p. 48. Note that there exist univariate distributions that are no variance mixtures of normals that can be marginals of spherical distributions for some, but not all $n \in \mathbf{N}$.

17 Note that any mixture of normals has a density with respect to the Lebesgue measure.

$$w = \frac{n}{\vartheta}$$

where $\vartheta \sim \chi_n^2$.

Example 2

A more flexible family of mixture distributions is the generalized hyperbolic distribution. The one-dimensional centered and symmetric version of the generalized hyperbolic distribution has three free parameters λ, α, δ and is defined by its Lebesgue-density

$$gh(x;\lambda,\alpha,\delta) = \mathrm{a}(\lambda,\alpha,\delta) \cdot \left(\delta - x^2\right)^{(\lambda-1/2)/2} \mathbf{K}_{\lambda-1/2}\left(\alpha\sqrt{\delta^2 + x^2}\right)$$

with

$$\mathrm{a}(\lambda,\alpha,\delta) = \frac{\alpha^{1/2}}{\sqrt{2\pi} \cdot \delta^{\lambda} \cdot \mathbf{K}_{\lambda}(\delta\lambda)}$$

where $\mathbf{K}_{\lambda}(\cdot)$ is a modified Bessel function of the third kind with index λ and $x \in \mathbf{R}$. Alternatively, the generalized hyperbolic distribution can be defined by its mixing distribution. This is the generalized inverse Gauss distribution with density

$$gig(x;\psi,\chi) = \frac{(\psi/\chi)^{\lambda/2}}{\mathbf{K}_{\lambda}\left(\sqrt{\psi\chi}\right)} x^{\lambda-1} \cdot \exp\left(-\frac{1}{2}\left(\frac{\chi}{x} + \psi x\right)\right)$$

for $x > 0$ and $\chi = \delta^2$ and $\psi = \alpha^2$.

The generalized hyperbolic distribution is continuous in its parameters and has the normal and the t-distribution as limiting cases:

For $\alpha, \delta \to \infty$, $\frac{\delta}{\alpha} \to \sigma^2$ and any given λ, the generalized hyperbolic distribution converges towards $\mathrm{N}\left(0,\sigma^2\right)$.[18]

On the other hand, for $\alpha = 0$, $\delta = \sqrt{\nu}$ and $\lambda = -\nu/2$ it is equal to the t-distribution with ν degrees of freedom.[19]

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[18] Cf. Prause (1999), p. 3.

[19] Cf. Prause (1999), p. 5.

Example 3

An interesting special case of the generalized hyperbolic distribution is the normal inverse Gauss distribution (NIG). It is obtained for $\lambda = -1/2$ and has the inverse Gauss distribution

$$ig(x;\psi,\chi) = \sqrt{\frac{\chi}{2 \cdot \pi \cdot x^3}} \cdot \exp\left(-\frac{1}{2}\left(\frac{\chi}{x} + \psi x \right) + \sqrt{\psi\chi} \right)$$

as mixing distribution.

The NIG is particularly interesting as an alternative for the normal distribution in the asset value model because it is not only infinitely divisible[20], but also closed under convolution. Hence, similar to the normal distribution, it generates a Lévy-motion whose finite dimensional marginals are all NIG-distributed[21]. Therefore, the intuitive interpretation in the Vasicek-Kealhofer model and in Credit Metrics of the normal distribution as the marginal distribution of an asset return process could be maintained in a one-to-one fashion if the normal distribution was replaced by the NIG.

Example 4

A very simple family of mixtures of normals are finite mixture distributions. They are obtained if the mixing distribution W takes on only finitely many values with positive probability, i.e. if there exist real numbers $w_1,\dots,w_n > 0$ such that

$$\mathbf{P}\left(w \in \{w_1,\dots,w_n\}\right) = 1$$

if $w \sim W$.

Besides the flexibility of deformation, the distributions differ above all in tail behavior.

[20] As is any generalized hyperbolic distribution. Cf. Barndorff-Nielsen and Halgreen (1977).

[21] Cf. Eberlein et al. (1998), p. 6f., who use the Lévy-motion generated by the NIG instead of the classical geometric Brownian motion to model financial price processes.

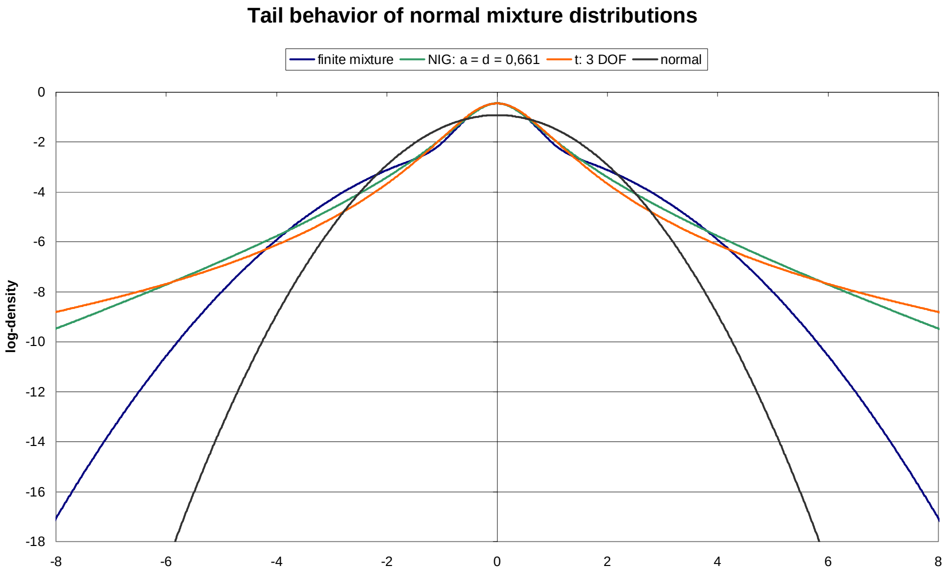

Figure 1: Tail behavior of normal mixture distributions

Figure 1 shows that NIG and t-distributions have exponentially decreasing tails while tails of finite mixture distributions and the normal distribution decrease of order $O\left(e^{-x^2}\right)$.

For a further interpretation of the illustration above, we need

Definition 3:

Let *f* be the density function of a distribution *F* with expectation μ and variance σ^2. We say that *F* has long tails compared to the normal distribution if and only if

$$\lim_{x\to\infty}\frac{f(x)}{\varphi_{\mu,\sigma^2}(x)}>1$$

where φ_{μ,σ^2} is the density of the normal distribution with expectation μ and variance σ^2.

We can now prove

Theorem 1:

Let *D* be a normal variance mixture with non-degenerate mixing distribution. Then *D* has long tails compared to the normal distribution.

Definition 4:

We say that a model defines a representation of the generalized correlation model, if its risk index distribution is a variance mixture of normals. According to the choice of the risk index distribution in the generalized correlation model, we will also speak of the Student-t-correlation model, the finite mixture correlation model etc.

The normal correlation model is just a special case in this framework. It is obtained for $w \equiv c$ for some constant $c > 0$.

To better understand the properties of the generalized correlation model and to see the impact of the choice of clients' risk index distributions on the resulting portfolio risk, in the following we look at homogenous portfolios because we can derive analytic loss distributions for this special class of portfolios, which render it possible to prove results.

We define a homogenous portfolio as consisting only of identical clients in terms of probabilities of default p, exposure E, risk index correlations ρ, and expected loss given default λ[22]. Without loss of generality, we assume that all risk index distributions are centered and have variance 1.

Let D be the distribution of risk indices. We assume that D is a mixture of normals with mixing distribution W, such that

$$\sqrt{w} \cdot X \sim D$$

where

$$w \sim W \text{ and } X \sim \mathrm{N}(0,1).$$

In the generalized correlation model in a homogenous portfolio each client's risk index X_i is then given as

$$X_i = \sqrt{w} \cdot \left(\sqrt{\rho} \cdot Y + \sqrt{1-\rho} \cdot Z_i\right)$$

with $i = 1,\dots,n$ if n is the number of clients in the portfolio where Y and Z_i are independent and standard normally distributed.

To facilitate the exposition of the results,

22 We do not assume recovery rates or loss given default rates as being fixed. They may be random with the same mean (not necessarily the same distribution) being independent from all other random variables in the model such as systematic and idiosyncratic risk factors.

- let F be the cumulative distribution function of D,
- let Φ be the cumulative distribution function of the standard normal distribution, and
- let L be the loss distribution of the portfolio under consideration, i.e. the cumulative distribution function of portfolio losses.

We begin with the derivation of the portfolio loss distribution of homogenous portfolios in the generalized correlation model, then calculate the density of the portfolio loss distribution and, finally, compare the results for the normal and generalized correlation model.

(1) Portfolio loss distribution

Theorem 2:

In the generalized correlation model in a homogenous portfolio containing n clients each having an exposure of $E = 1/n$ and risk index correlations $0 < \rho < 1$ the asymptotic portfolio loss distribution is given as

$$\begin{aligned} L(l; p, \rho, \lambda) &= \lim_{n \to \infty} L(l; p, E(n), \rho, \lambda) = \mathbf{P}\{\text{Loss} \le l\} \\ &= \mathbf{E}_w \left(\Phi\left(\frac{\sqrt{w} \cdot \sqrt{1-\rho} \cdot \Phi^{-1}(l/\lambda) - F^{-1}(p)}{\sqrt{w} \cdot \sqrt{\rho}} \right) \right) \end{aligned}$$

where $\mathbf{E}_w(\cdot)$ is the expectation functional with respect to w.

From Theorem 2 we obtain the loss distribution in the normal correlation model for $w \equiv 1$, i.e.

$$L(l; p, \rho, \lambda) = \Phi\left(\frac{\sqrt{1-\rho} \cdot \Phi^{-1}(l/\lambda) - \Phi^{-1}(p)}{\sqrt{\rho}} \right),$$

a result that was already proved by Vasicek (1991).

The portfolio loss distribution is generally easy to calculate for finite mixture distributions because in this case the expectation functional $\mathbf{E}_w(\cdot)$ is reduced to a simple sum. Let the mixing distribution W be a probability distribution on $\{w_1, \ldots, w_k\}$ with $\mathbf{P}\{w = w_i\} = p_i$ for $i = 1, \ldots, k$. Then the portfolio loss distribution can be written as

$$L(l; p, \rho, \lambda) = \sum_{i=1}^{k} p_i \cdot \Phi\left(\frac{\sqrt{w_i} \cdot \sqrt{1-\rho} \cdot \Phi^{-1}(l/\lambda) - F^{-1}(p)}{\sqrt{w_i} \cdot \sqrt{\rho}} \right).$$

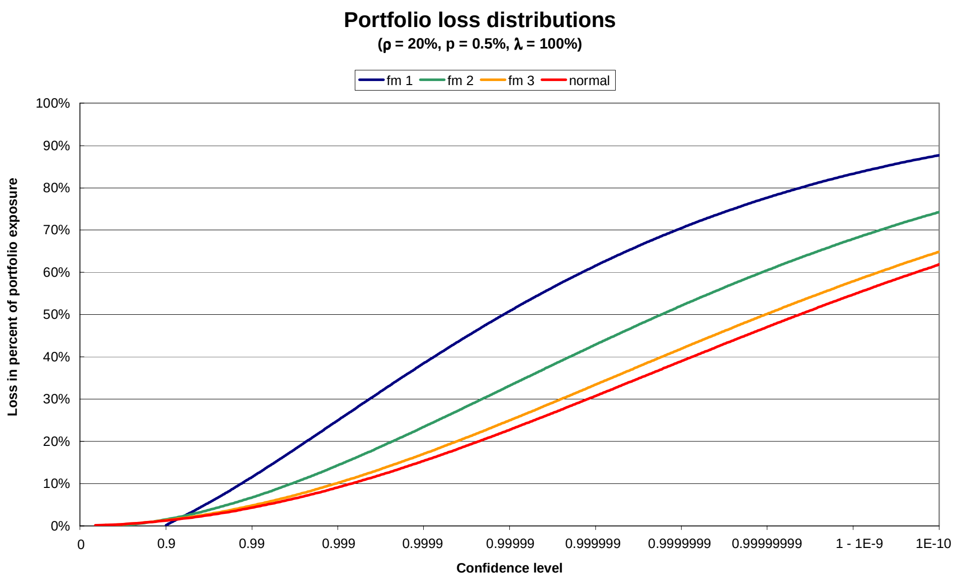

Figure 2: Portfolio loss distributions in the generalized correlation model based on finite mixture distributions[23]

Figure 2 shows loss distributions for a homogenous portfolio resulting from the finite mixture correlation model. Note that the loss distributions in the finite mixture correlation model dominate the loss distribution in the normal correlation model at high confidence levels, in our above example at confidence levels above 92.5% (fm 1), 84.8% (fm 2), and 79.9% (fm 3). We will prove this as a general result in Theorem 6 below.

For more complex mixing distributions the portfolio loss distribution can be calculated using numerical techniques or the Monte Carlo integration.

23 The mixing distributions of the finite mixtures in the graph are defined as follows:

fm 1: $\mathbf{P}\{w = 0.35\} = 0.9, \quad \mathbf{P}\{w = 6.85\} = 0.1$

fm 2: $\mathbf{P}\{w = 0.35\} = 0.65, \quad \mathbf{P}\{w = 2.21\} = 0.35$

fm 3: $\mathbf{P}\{w = 0.35\} = 0.225, \quad \mathbf{P}\{w = 1.19\} = 0.775$

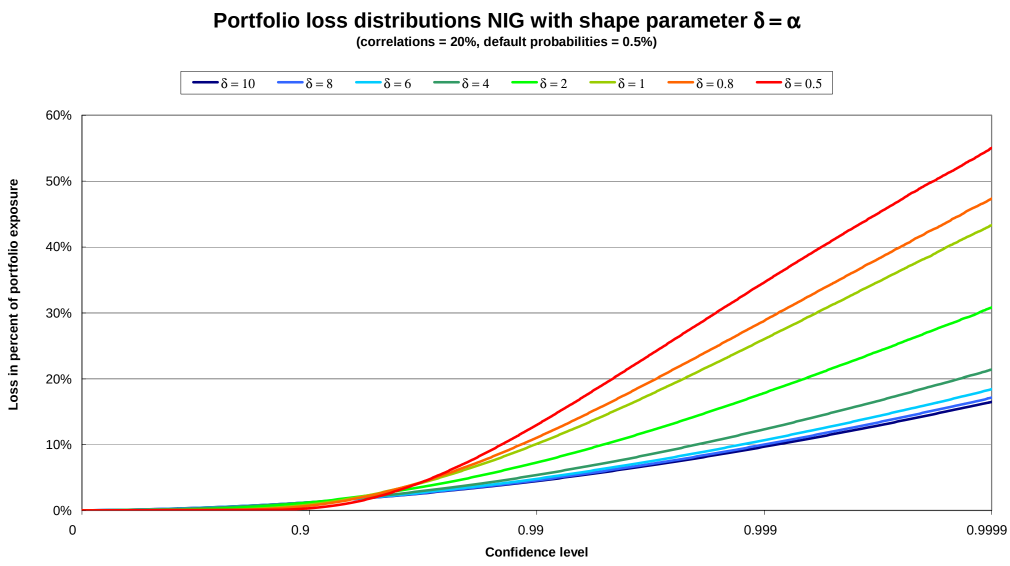

Figure 3: Portfolio loss distributions in the generalized correlation model based on normal inverse Gaussian distributions

It is a very important feature of the generalized correlation model that clients with uncorrelated risk indices are still dependent in their default behavior if the mixing distribution W is non-trivial. We state this fact as

Theorem 3:

In the generalized correlation model, in a homogenous portfolio containing n clients each having an exposure of $E = 1/n$ and risk index correlations $\rho = 0$, the asymptotic portfolio loss distribution is given as

$$L(l;p,0,\lambda) = \begin{cases} \mathbf{P}\left\{\sqrt{w} \le \dfrac{F^{-1}(p)}{\Phi^{-1}(l/\lambda)}\right\} & \text{if } l < \lambda/2 \\ 1 - \mathbf{P}\left\{\sqrt{w} \le \dfrac{F^{-1}(p)}{\Phi^{-1}(l/\lambda)}\right\} & \text{if } l > \lambda/2 \\ 0 & \text{if } p > 1/2 \text{ and } l = \lambda/2 \\ 1 & \text{if } p \le 1/2 \text{ and } l = \lambda/2 \end{cases}$$

Theorem 3 states that – other than in the normal correlation model[24] where $\rho = 0$ implies $\mathbf{P}\{\text{Loss} = p \cdot \lambda\} = 1$ – uncorrelated non-normal risk indices in the generalized correlation model imply a constant portfolio loss distribution only for p = ½.

[24] Again, the result for the normal correlation model follows from the theorem for $w \equiv 1$. In this case $F = \Phi$.

For default probabilities $p < ½$ the loss distribution is not constant, but only takes values between 0 and $\lambda/2$. On the other hand, for default probabilities $p > ½$ the loss distribution is not constant either, and only takes values between $\lambda/2$ and λ.

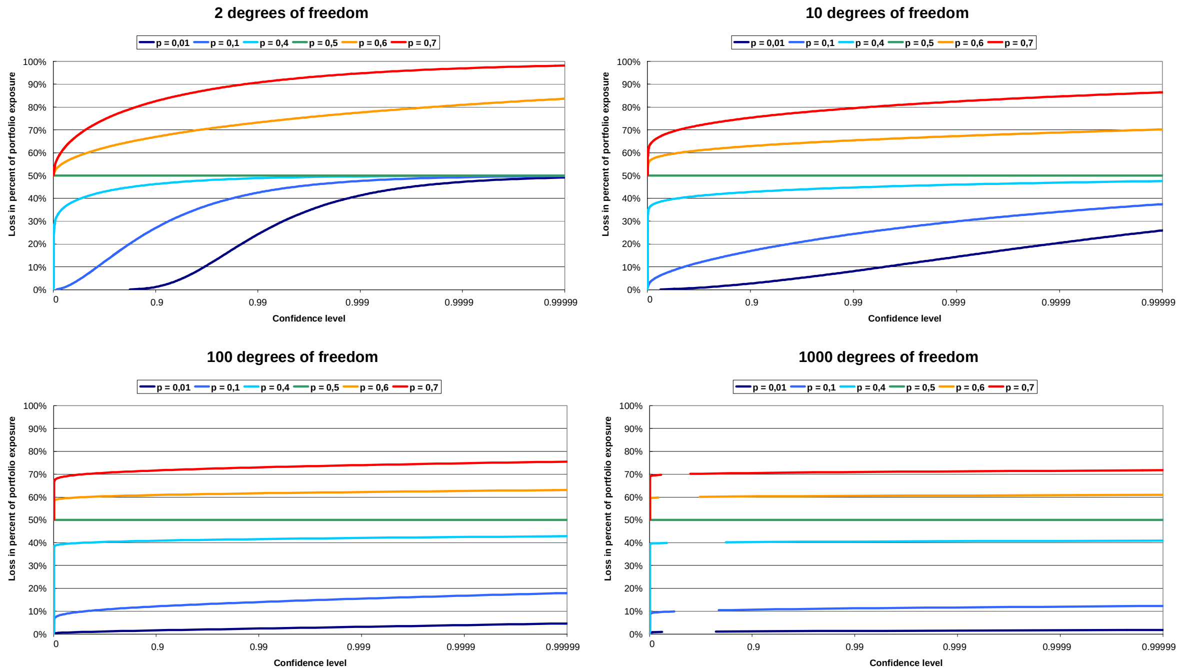

Figure 4: Loss distributions resulting from uncorrelated Student-t-distributed risk indices[25]

Figure 4 shows that loss distributions remain above or below $\lambda/2$ for the respective probabilities of default. If the number of the degrees of freedom of the Student-t-distribution tends towards infinity, i.e. if the t-distributed risk indices converge towards normally distributed risk indices, the portfolio loss distributions become more and more flat and converge towards $L(l; p, 0, \lambda) \equiv p \cdot \lambda$ as we would expect from Theorem 3.

If the mixing distribution of the risk index distributions is discrete, as is the case at finite mixtures of normals, then the resulting portfolio loss distribution is also discrete given the risk index correlations are zero (Figure 5). However, if the mixing distribution converges to a constant, the loss distribution again converges towards $L(l; p, 0, \lambda) \equiv p \cdot \lambda$.

[25] The loss given default rate λ is set to 100%.

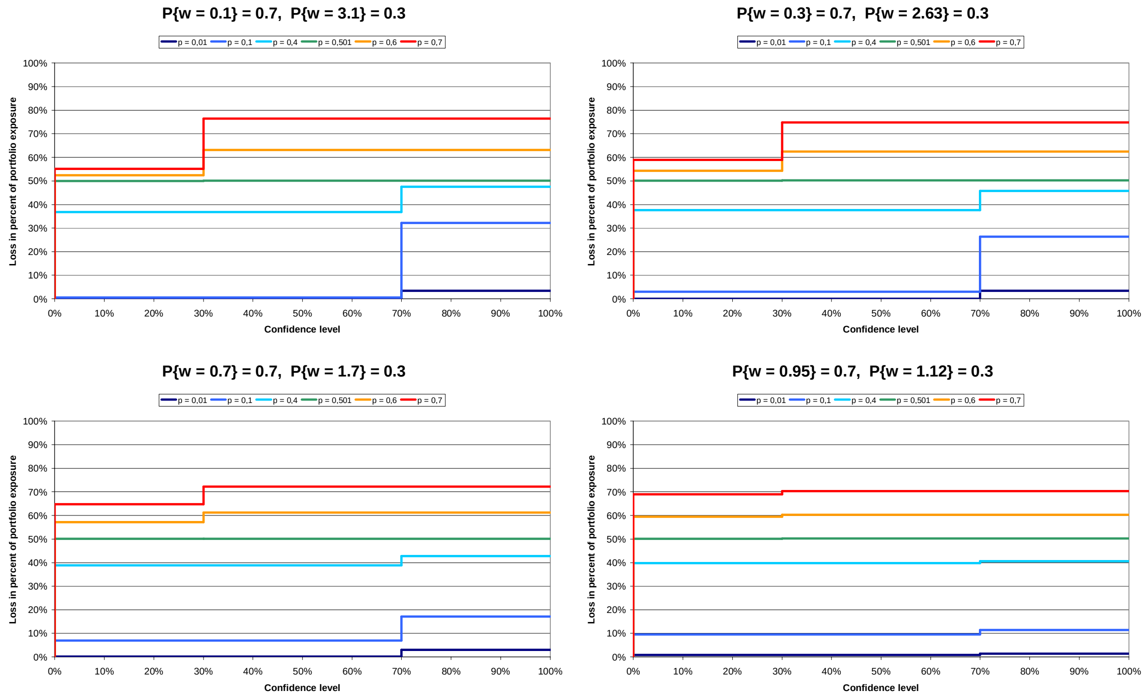

Figure 5: Loss distributions resulting from uncorrelated finite mixture distributed risk indices[26]

In the remaining case of perfect risk index correlations $\rho = 1$ the differences between the models disappear. Here clients' risk indices are given as

$$X_i = X = \sqrt{w} \cdot Y$$

so that all clients default simultaneously if

$$X = \sqrt{w} \cdot Y \leq F^{-1}(p).$$

However, since $\sqrt{w} \cdot Y \sim F$ it is

$$\mathbf{P}\left\{\sqrt{w} \cdot Y \leq F^{-1}(p)\right\} = \mathbf{P}\left\{Y \leq \Phi^{-1}(p)\right\} = p$$

independent of F.

(2) Portfolio loss density

In the previous section, we considered the asymptotic portfolio loss distribution of homogenous portfolios for an infinite number of clients in the portfolio. Due to the increasing number of clients, each single client's exposure converges to zero relative to the total portfolio expo-

[26] The mixing distribution is a two-point distribution that is standardized to have expectation 1. The loss given default rate λ is set to 100%.

sure. Therefore, homogenous portfolios asymptotically do not contain exposure concentrations on specific clients or exposure point masses. This is a necessary condition for the portfolio loss distribution in order to have a Lebesgue density.

In the next theorem, we derive the portfolio loss density for risk index correlations $0<\rho<1$.

Theorem 4:

In the generalized correlation model, in a homogenous portfolio containing *n* clients each having an exposure of $E = 1/n$ and risk index correlations $0<\rho<1$, the density L_d of the asymptotic portfolio loss distribution is given as

$$L_d\left(l;\ p,\rho,\lambda\right)=\frac{\sqrt{1-\rho}}{\lambda\cdot\sqrt{\rho}}\cdot\frac{1}{\varphi\left(\Phi^{-1}\left(l/\lambda\right)\right)}\cdot\mathbf{E}_w\left(\varphi\left(\frac{F^{-1}\left(p\right)-\sqrt{w}\cdot\sqrt{1-\rho}\cdot\Phi^{-1}\left(l/\lambda\right)}{\sqrt{w}\cdot\sqrt{\rho}}\right)\right)$$

where φ is the standard normal density.

Figure 6 shows loss densities for a model where the risk index distribution is a bi-mixture defined by its mixing distribution $\mathbf{P}\{w=0.2\}=0.5$ and $\mathbf{P}\{w=1.8\}=0.5$ for various default probabilities and risk index correlations.

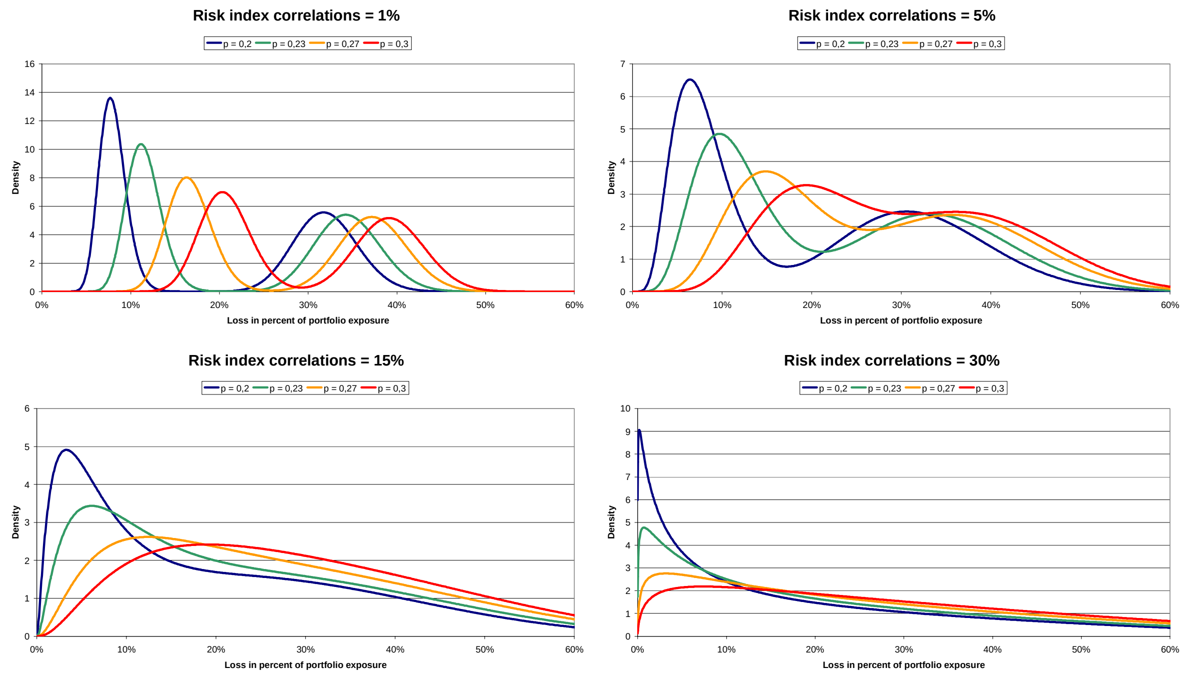

Figure 6: Portfolio loss densities in the bimixture correlation model

Note that the density function is not necessarily unimodal. The number of modes varies with default probabilities, with risk index correlations and also with the type of mixing distribution.

We formulate this observation as

Theorem 5:

In the generalized correlation model the number of modes of the asymptotic portfolio loss distribution of a homogenous portfolio with risk index correlations $0 < \rho < 1$ is smaller or equal to the cardinality of the support of the mixing distribution of the risk index distribution.

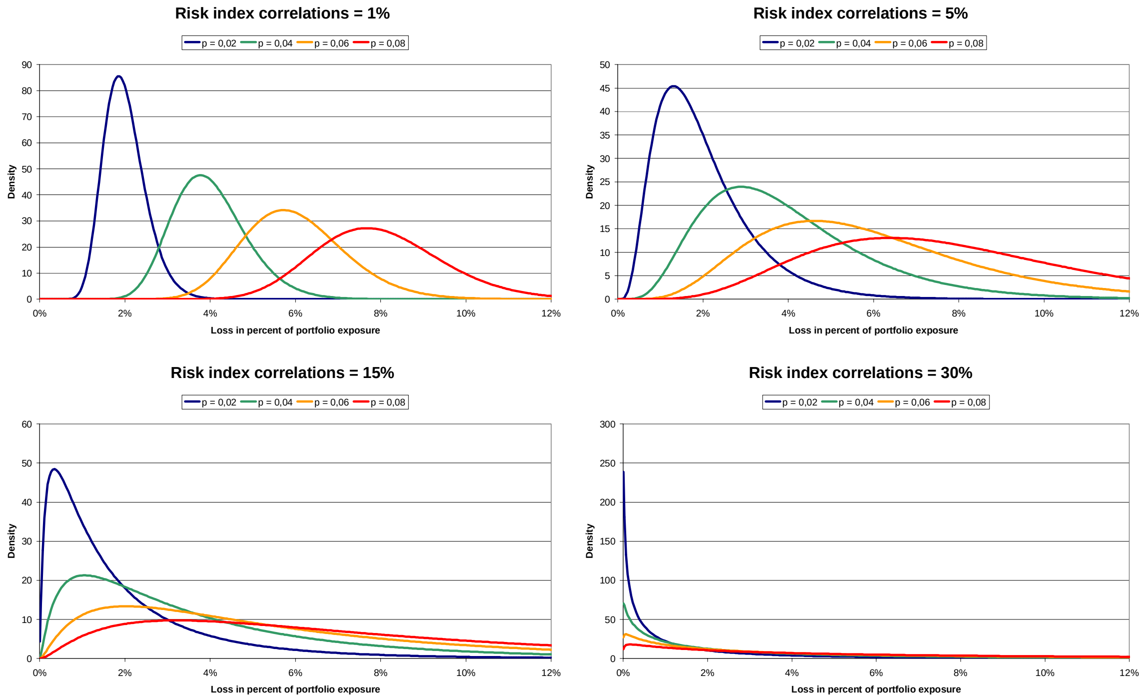

Figure 7: Portfolio loss densities in the normal correlation model

Figure 8 gives an example of trimodal loss densities in the trimixture correlation model.

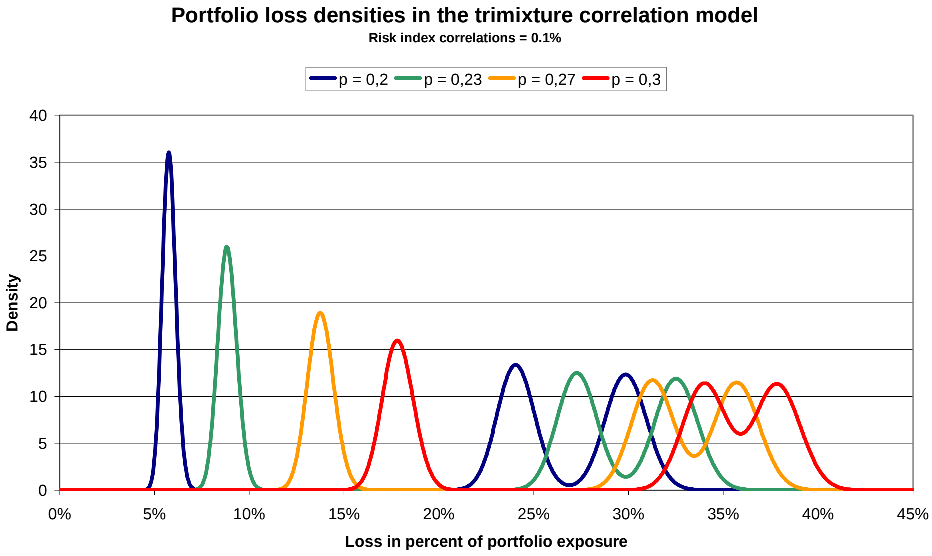

Figure 8: Trimodal portfolio loss densities in the trimixture correlation model[27]

We saw in the previous section, that the portfolio loss distribution is discrete for zero correlations in the finite mixture correlation model while it is Lebesgue absolutely continuous for positive risk index correlations. We would, therefore, expect that loss densities degenerate in this model if risk index correlations go to zero, a phenomenon that is illustrated in Figure 9.

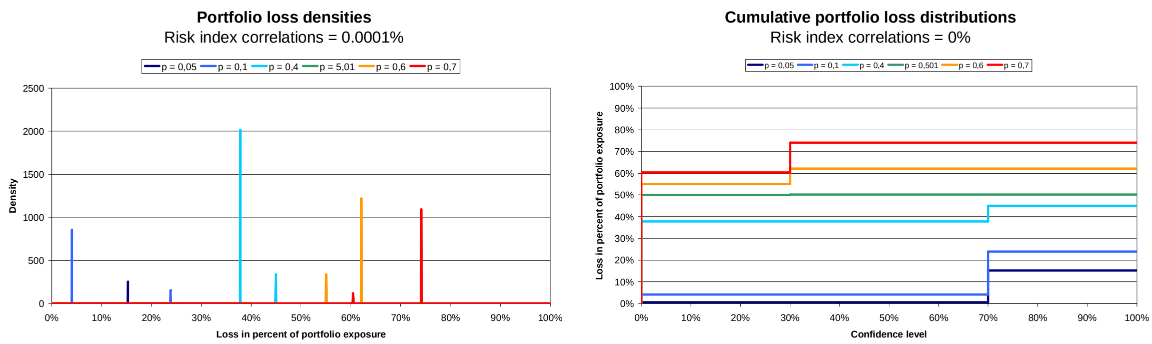

Figure 9: The low correlation effect in the finite mixture model[28]

For extremely low correlations, the loss density develops peaks at the discontinuity points of the loss distribution. The relative size of the peaks is approximately equal to the size of the point masses at the discontinuity points.

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[27] The distributions differ only in the clients' default probability p and share the mixing distribution $\mathbf{P}\{w = 0.2\} = 1/3$, $\mathbf{P}\{w = 1\} = 1/3$, $\mathbf{P}\{w = 1.78\} = 1/3$. The modes of the original loss densities in the convex combinations are well visible and well separated.

[28] The actual model displayed in the charts is a bimixture model with the mixing distribution $\mathbf{P}\{w = 0.4\} = 0.7$ and $\mathbf{P}\{w = 2.4\} = 0.3$.

(3) Comparison of the normal and the generalized correlation model

So far the replacement of the normal distribution by a general distribution, determined by its marginal distributions and a correlation structure, was rather intuitively motivated as a general topic in the correlation model and by the fact that all of these distributions have long tails.

The next theorem gives a fundamental reason as to why the choice of the normal distribution as the distribution of risk indices might cause structural problems in the asset value model and why it should be carefully overthought. The theorem also shows that the analysis of homogenous portfolios can be extremely helpful in understanding the economic and model theoretic consequences of allegedly natural mathematical assumptions.

Theorem 6:

Let $L_N^{-1}(\cdot\,;p,\rho,\lambda)$ and $L_{G\setminus N}^{-1}(\cdot\,;p,\rho,\lambda)$ be the inverse cumulative distribution functions of the asymptotic portfolio loss distributions of a homogenous portfolio with risk index correlations $0<\rho<1$ in the generalized correlation model with a normal and a non-normal risk index distribution respectively. Then there exists a confidence level α^* such that

$$L_{G\setminus N}^{-1}(\alpha;p,\rho,\lambda) > L_N^{-1}(\alpha;p,\rho,\lambda)$$

for all confidence levels $\alpha>\alpha^*$ and $p\neq\frac{1}{2}$.

Alternatively: Let $L_N(\cdot\,;p,\rho,\lambda)$ and $L_{G\setminus N}(\cdot\,;p,\rho,\lambda)$ be the cumulative distribution functions of the asymptotic portfolio loss distributions of the same homogenous portfolio with risk index correlations $0<\rho<1$ in the generalized correlation model with a normal and a non-normal risk index distribution respectively. Then there exists a portfolio loss l^* such that

$$L_{G\setminus N}(l;p,\rho,\lambda) < L_N(l;p,\rho,\lambda)$$

for all portfolio losses $l>l^*$.

Theorem 6 states that for high confidence levels the normal correlation model reports less risk in any homogenous portfolio than any other correlation model (see Figure 10). I.e. at the high confidence levels risk managers are interested in, the normal correlation model is not robust against misspecifications in the risk index distribution.

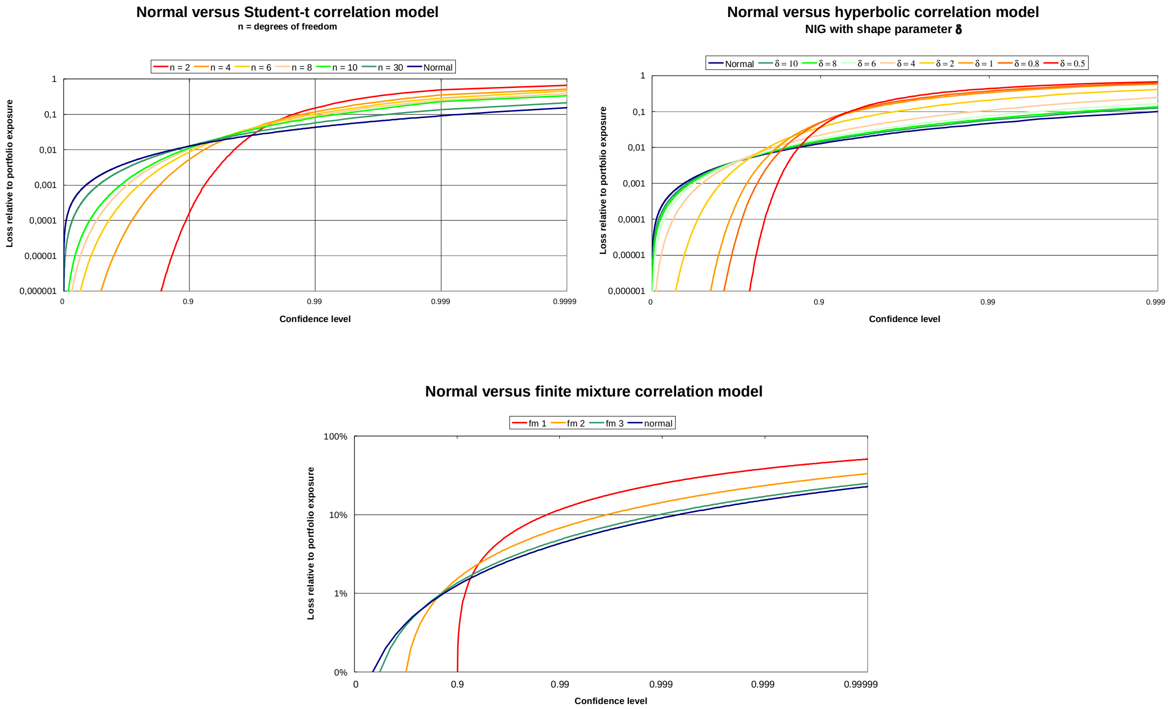

Figure 10: Portfolio loss distributions in the normal versus the generalized correlation model

It is worth noting that Theorem 6 also proves that the property of tail dependence, shown by some multivariate distributions, is not relevant for a specific correlation model in order to detect a higher portfolio risk than the normal correlation model at high percentiles[29].

Tail dependence is an asymptotic measure of dependence of bivariate distributions that is often used to describe dependence of extreme events.

Definition 5:[30]

Let X and Y be random variables with distribution functions F_1 and F_2. The coefficient of upper tail dependence of X and Y is

$$\lim_{\alpha \to 1-} \mathbf{P}\left\{ Y > F_2^{-1}(\alpha) \mid X > F_1^{-1}(\alpha) \right\} = \lambda$$

provided a limit $\lambda \in [0, 1]$ exists. If $\lambda \in (0, 1]$, X and Y are said to be asymptotically dependent in the upper tail. If $\lambda = 0$, X and Y are said to be asymptotically independent.

29 This disproves a hypotheses by Nyfeler (2000), p. 50ff., Frey and McNeil (2001), p. 16, and Frey et al. (2001), p. 5ff., who in simulation experiments found that the multivariate t-distribution as risk index distribution led to higher portfolio risk than the normal distribution and blamed this observation on the tail dependence property of the t-distribution.

30 See Schmidt (2002), p. 302, definition 1.1 or Embrechts et al. (1999), p. 18, definition 7.

Theorem 6 states that the normal correlation model is risk minimal among all correlation models with elliptical risk index distributions. It can, however, be shown that some multivariate elliptical distributions are tail independent as, for instance, the logistic distributions and the symmetric hyperbolic distributions which also include the NIG[31].

Conclusion

We have defined the generalized correlation model as an extension of the classical asset value credit risk model that is widely used in the banking sector worldwide. For homogenous portfolios, we have derived a number of properties of the wider family of models. As our main result, we have shown that the normal correlation model – the classical asset value model – is risk minimal within the generalized correlation framework.

This shows that not only the underlying portfolio but also the superimposed modeling assumptions do significantly influence the reported portfolio risk and that a model can be highly sensitive to misspecifications possibly leading to insufficient risk management actions. It particularly implies that financial institutions using the normal correlation model for risk assessment might underestimate their true credit portfolio risk.

[31] See Schmidt (2002), p. 324f., theorem 6.6 and theorem 6.8.

Appendix

Proof of Theorem 1:

Without loss of generality, we can assume that $\mu = 0$ and $\sigma^2 = 1$. Let *W* be the mixing distribution of *D* and $w \sim W$. Then there exists an *s* > 1 such that $p := \mathbf{P}\{w \geq s\} > 0$.

Let f be the density of *D*. Then we have

$$\lim_{x\to\infty} \frac{f(x)}{\varphi(x)} > \lim_{x\to\infty} \frac{p \cdot \frac{1}{\sqrt{s}} \cdot \varphi\left(\frac{x}{\sqrt{s}}\right)}{\varphi(x)} = \lim_{x\to\infty} \frac{p \cdot e^{-\frac{x^2}{2s}}}{\sqrt{s} \cdot e^{-\frac{x^2}{2}}} = \lim_{x\to\infty} \frac{p}{\sqrt{s}} e^{\frac{x^2}{2}\left(1-\frac{1}{s}\right)} = \infty .$$

□

Proof of Theorem 2:

By definition of the model client *i* defaults if

$$X_i = \sqrt{w} \cdot \left(\sqrt{\rho} \cdot Y + \sqrt{1-\rho} \cdot Z_i\right) \leq F^{-1}(p)$$
$$\Leftrightarrow Z_i \leq \frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{\rho} \cdot Y}{\sqrt{w} \cdot \sqrt{1-\rho}}$$

for $i = 1,...,n$.

Hence, client *i*'s probability of default conditional to *w* and *Y* is given as

$$\mathbf{P}\{\text{client } i \text{ defaults} \mid w, Y\} = \Phi\left(\frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{\rho} \cdot Y}{\sqrt{w} \cdot \sqrt{1-\rho}}\right)$$

because Z_i is standard normally distributed for $i = 1,...,n$.

Moreover, since the idiosyncratic components Z_i of clients' risk indices are stochastically independent, it follows from the law of large numbers that the percentage of clients defaulting in the portfolio given *w* and *Y* is equal to their conditional probability of default with probability one if $n \to \infty$.

Note that asymptotically the number of defaulting clients goes towards infinity as well if the conditional probability is positive. Thus, again by the law of large numbers, the portfolio loss conditional to *w* and *Y* is equal to

$$\text{Loss} \mid w, Y = \lambda \cdot \Phi\left(\frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{\rho} \cdot Y}{\sqrt{w} \cdot \sqrt{1-\rho}}\right)$$

since the individual loss given default rates are stochastically independent and limited with the same mean λ.

The unconditional portfolio loss distribution is, therefore, given as

$$\begin{aligned}
\mathbf{P}\{\text{Loss} \le l\} &= \mathbf{P}\left\{\lambda \cdot \Phi\left(\frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{\rho} \cdot Y}{\sqrt{w} \cdot \sqrt{1-\rho}}\right) \le l\right\} \\
&= \mathbf{P}\left\{\frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{\rho} \cdot Y}{\sqrt{w} \cdot \sqrt{1-\rho}} \le \Phi^{-1}(l/\lambda)\right\} \\
&= \mathbf{P}\left\{Y \ge \frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{1-\rho} \cdot \Phi^{-1}(l/\lambda)}{\sqrt{w} \cdot \sqrt{\rho}}\right\} \\
&= 1 - \mathbf{P}\left\{Y \le \frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{1-\rho} \cdot \Phi^{-1}(l/\lambda)}{\sqrt{w} \cdot \sqrt{\rho}}\right\} \\
&= 1 - \mathbf{E}_w\left(\mathbf{P}\left\{Y \le \frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{1-\rho} \cdot \Phi^{-1}(l/\lambda)}{\sqrt{w} \cdot \sqrt{\rho}} \mid w\right\}\right) \\
&= 1 - \mathbf{E}_w\left(\Phi\left(\frac{F^{-1}(p) - \sqrt{w} \cdot \sqrt{1-\rho} \cdot \Phi^{-1}(l/\lambda)}{\sqrt{w} \cdot \sqrt{\rho}}\right)\right)
\end{aligned}$$

The theorem then follows from $1 - \Phi(x) = \Phi(-x)$.

□

Proof of Theorem 3:

If risk indices are uncorrelated, client *i* defaults if

$$\begin{aligned}
&X_i = \sqrt{w} \cdot Z_i \le F^{-1}(p) \\
&\Leftrightarrow Z_i \le \frac{F^{-1}(p)}{\sqrt{w}}
\end{aligned} \quad .$$

Along the same lines as in Theorem 2 one can show that the portfolio loss distribution is then given as

$$\begin{aligned}
\mathbf{P}\{\text{Loss} \le l\} &= \mathbf{P}\left\{\lambda \cdot \Phi\left(\frac{F^{-1}(p)}{\sqrt{w}}\right) \le l\right\} \\
&= \mathbf{P}\left\{\frac{F^{-1}(p)}{\sqrt{w}} \le \Phi^{-1}(l/\lambda)\right\} \\
&= \mathbf{P}\left\{F^{-1}(p) \le \sqrt{w} \cdot \Phi^{-1}(l/\lambda)\right\}
\end{aligned}$$

which is equivalent to the formulation in the theorem since $\Phi^{-1}(l/\lambda) < 0$ if $l < \lambda/2$, $\Phi^{-1}(l/\lambda) > 0$ if $l > \lambda/2$, and $\Phi^{-1}(l/\lambda) = 0$ if $l = \lambda/2$.

□

Proof of Theorem 4:

The portfolio loss density is defined as the first derivative of the cumulative distribution function of portfolio losses. Thus, we have

$$\begin{aligned}
L_d(l;p,\rho,\lambda) &= \frac{d}{d\,l}L(l;p,\rho,\lambda) \\
&= \frac{d}{d\,l}\left(1-\mathbf{E}_w\left(\Phi\left(\frac{F^{-1}(p)-\sqrt{w}\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)}{\sqrt{w}\cdot\sqrt{\rho}}\right)\right)\right) \\
&= -\mathbf{E}_w\left(\varphi\left(\frac{F^{-1}(p)-\sqrt{w}\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)}{\sqrt{w}\cdot\sqrt{\rho}}\right)\cdot(-1)\cdot\frac{\sqrt{w}\cdot\sqrt{1-\rho}}{\sqrt{w}\cdot\sqrt{\rho}}\cdot\frac{1}{\varphi\left(\Phi^{-1}(l/\lambda)\right)}\cdot\frac{1}{\lambda}\right)
\end{aligned}$$

since the standard normal density is continuous and integrable so that we may derive within the expectation functional $\mathbf{E}_w$.

□

Proof of Theorem 5:

It can be shown by derivation of the portfolio loss density of the normal correlation model[32], that the portfolio loss distribution is unimodal for $0<\rho<1/2$ in this model (see also Figure 7). For $1/2<\rho<1$ the portfolio loss density is bimodal with peaks at 0 and the maximum loss.

The theorem then follows immediately from the fact that the portfolio loss distribution in the generalized correlation model is the convex combination of loss distributions in the normal correlation model.

□

Proof of Theorem 6:

Let $L_{d,N}(\cdot\,;p,\rho,\lambda)$ and $L_{d,G\backslash N}(\cdot\,;p,\rho,\lambda)$ be the portfolio loss densities in the correlation model with a normal and a non-normal risk index distribution. We show that a portfolio loss l^* exists so that $L_{d,N}(l\,;p,\rho,\lambda)<L_{d,G\backslash N}(l;p,\rho,\lambda)$ for all portfolio losses $l>l^*$ (see Figure 11). This implies that $L_{G\backslash N}(l;p,\rho,\lambda)<L_N(l;p,\rho,\lambda)$ and, thus, the second version of the theorem because $L_{G\backslash N}(1;p,\rho,\lambda)=L_N(1;p,\rho,\lambda)=\lambda$.

[32] The portfolio loss density of the normal correlation model is obtained by setting $w\equiv c$ for some constant c in Theorem 4.

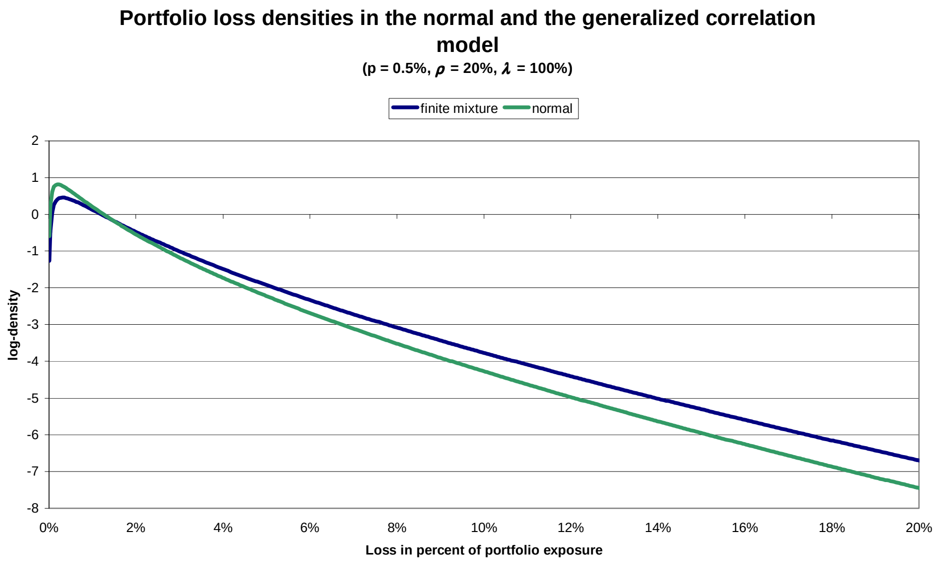

Figure 11: Portfolio densities in the normal and the generalized correlation model

Theorem 4 implies that

$$\frac{L_{d,G\backslash N}(l\,;p,\rho,\lambda)}{L_{d,N}(l\,;p,\rho,\lambda)}=\mathbf{E}_W\left(\frac{\varphi\left(\dfrac{F^{-1}(p)-\sqrt{w}\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)}{\sqrt{w}\cdot\sqrt{\rho}}\right)}{\varphi\left(\dfrac{\Phi^{-1}(p)-\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)}{\sqrt{\rho}}\right)}\right)$$

$$=\mathbf{E}_W\left(\exp\left(-\frac{1}{2}\cdot\left(\frac{F^{-1}(p)-\sqrt{w}\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)}{\sqrt{w}\cdot\sqrt{\rho}}\right)^2+\frac{1}{2}\cdot\left(\frac{\Phi^{-1}(p)-\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)}{\sqrt{\rho}}\right)^2\right)\right)$$

$$=\mathbf{E}_W\left(\exp\left(\frac{1}{2w\rho}\cdot\left(\underbrace{w\left[\Phi^{-1}(p)\right]^2-\left[F^{-1}(p)\right]^2}_{=:T_1}\right.\right.\right.$$

$$\left.\left.\left.+\underbrace{2\sqrt{w}\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)\cdot F^{-1}(p)-2w\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)\cdot\Phi^{-1}(p)}_{=:T_2}\right)\right)\right)$$

The statement $L_{d,N}(l\,;p,\rho,\lambda)<L_{d,G\backslash N}(xl;p,\rho,\lambda)$ is equivalent to $\dfrac{L_{d,G\backslash N}(l\,;p,\rho,\lambda)}{L_{d,N}(l\,;p,\rho,\lambda)}>1$.

Note that the term T_1 is constant for given *w*.

$$\begin{aligned}T_2&=2\sqrt{w}\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)\cdot F^{-1}(p)-2w\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)\cdot\Phi^{-1}(p)\\&=2\sqrt{w}\cdot\sqrt{1-\rho}\cdot\Phi^{-1}(l/\lambda)\cdot\left(F^{-1}(p)-\sqrt{w}\cdot\Phi^{-1}(p)\right)\end{aligned}$$

Since $\Phi^{-1}(l/\lambda)\to\infty$ for $l\to\lambda$ and $\exp(-x)\to 0$ for $x\to\infty$, the theorem holds if

$$\mathbf{P}\{F^{-1}(p)-\sqrt{w}\cdot\Phi^{-1}(p)>0\}=\mathbf{P}\{F^{-1}(p)>\sqrt{w}\cdot\Phi^{-1}(p)\}>0\,. \qquad (*)$$

Let $S := S_w = [w_{\min}; w_{\max}]$ be the support of the mixing distribution *W*.

Let $p < ½$. If $w_{\max}=\infty$, (*) holds trivially because $\Phi^{-1}(p)<0$.

If $w_{\max}<\infty$, it follows from the continuity and symmetry of *F* that

$$F^{-1}(p)=\inf\left\{l<0:\int_{\mathbf{R}^{>0}}\int_0^l\frac{1}{\sqrt{w}}\varphi\left(\frac{z}{\sqrt{w}}\right)dz\,W(dw)=p\right\}$$
$$=\inf\left\{l<0:\int_{\mathbf{R}^{>0}}\Phi\left(\frac{l}{\sqrt{w}}\right)W(dw)=p\right\}$$
$$>\inf\left\{l<0:\Phi\left(\frac{l}{\sqrt{w_{\max}}}\right)=p\right\}=:\tilde{l}=\sqrt{w_{\max}}\cdot\Phi^{-1}(p)$$

since the mixing distribution *W* is non-degenerate by assumption and $l < 0$.

Hence, there exists $\tilde{w}\in S$ such that $\mathbf{P}\left\{w_{\max}\geq w\geq\tilde{w}\right\}>0$ and

$$\mathbf{P}\{F^{-1}(p)>\sqrt{w}\cdot\Phi^{-1}(p)\}>0\,.$$

The case $p > ½$ can be solved with an analogous argument.

□

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