Dynamic Value-at-Risk

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Contest

1. Introduction

In the financial world nowadays, Value-at-Risk has become one of the most important if not the most used measures of risk. The models to negotiate portfolio risk were developed very quickly from the traditional distribution of profit and loss to dynamic Value-at-Risk. As a risk-management technique Value-at-Risk describes the loss that can occur over a given period, at a given confidence level, due to exposure to market risk. The simplicity of the Value-at-Risk concept has directed many to recommend that it become a standard risk measure, not only for financial establishments involved in large-scale trading operations, but also for retail banks, insurance companies, institutional investors and non-financial enterprises. As we see, Value-at-Risk has become an inalienable tool for risk control and an integral part of methodologies that dispense of capital between various business spheres. Its use is being encouraged by the Bank for International Settlements, the American Federal Reserve Bank and the Securities and Exchange Commission for every derivatives user. Today we can even speak about Portfolio Value-at-Risk. A major problem of the Value-at-Risk concept, however, is that it is calculated statically without analysis of the daily changes of surrounding financial, economic and social conditions.

The main objective of portfolio management is optimisation of asset allocation according to expected returns and risk class. It is rather difficult to compare various portfolio management strategies with the different instrument types. So one needs an unique and universal risk measurement tool. One of the modern and widely used risk measures is the Value-at-Risk, which helps to manage in the first line market risk. Customers would like to know about possible losses in their portfolio under the certain suggestions. Today we can find a lot of risk estimation methods, which let us measure risk in figures. Nevertheless, the general problem for them is static estimation without adaptation to the surrounding financial and economic conditions. That is why the building of dynamic strategy of the Value-at-Risk estimation are very effective.

Every portfolio can be characterised by positions on a certain number of risk factors. As we can estimate the Value-at-Risk for single financial instruments, we can add the all possible losses to express portfolio Value-at-Risk. The Value-at-Risk of a portfolio can be reconstructed from the combination of the risks of underlying securities. The purpose of this study is to describe dynamic Value-at-Risk and to estimate the advantages and disadvantages of using it in portfolio management. As a result one would like to present empirical studies with a comparison of the different methods and models of Value-at-Risk that are variously used in the banking sector.

2. Risk Measurement in Portfolio Management

2.1. Definition of the problem

Since Value-at-Risk received its first wide representation in July 1993 in the Group of Thirty report, the number of users of – and uses for – Value-at-Risk have increased dramatically. But it is important to recognise that the Value-at-Risk technique has gone through significant refinement and passed essential process changes since it originally appeared. Research in the field of financial economics has long distinguished the importance of measuring the risk of portfolio and financial assets or securities. Indeed, concerns go back at least five decades, when Markowitz's pioneering work on portfolio selection (1959) investigated the appropriate definition and measurement of risk. In recent years, the growth of trade activity and instances of financial market instability have prompted new studies underscoring the need for market participation to develop reliable risk measurement techniques.

Theoretical research that relied on the Value-at-Risk as a risk measurement was initiated by Jorion (1997), Dowd (1998), and Saunders (1999), who applied the Value-at-Risk approach based on risk management emerging as the industry standard by choice or by regulation. Value-at-Risk – based management by financial as well as non-financial institutions was researched and described by Bodnar (1998). Its wide useoccurs from the fact that Value-at-Risk is an easily interpretable summary measure of risk and also has an attractive explanation, as it allows its users to focus attention on the so-called "normal market condition" in their routine operations. Value-at-Risk models aggregate the several components of price risk into a single quantitative measure of the potential for losses over a specified time horizon. These models are clearly appealing because they convey the market risk of the entire portfolio in one number. Moreover, Value-at-Risk measures focus directly, and in currency (frank, dollar, rubl etc.) terms, on a major reason for assessing risk in the first place – a loss of portfolio value.

Recognition of such models by the financial and regulatory communities is evidence of their growing use. Since 1997 the Securities and Exchange Commission has required different financial structures, including banks and other large-capitalisation registrants, to quantify and report their market-risk exposure with Value-at-Risk disclosure being one measure in order to comply. So regulators also view Value-at-Risk as an useful summary measure. The risk-monitoring facet of Value-at-Risk is encouraged by regulators as well. In a risk-based capital proposal (1996), the Basle Committee on Banking Supervision endorsed the use of Value-at-Risk models, contingent on important qualitative and quantitative standards. In addition, the Bank for International Settlements Fisher report (1994) assisted in promoting the use of the Value-at-Risk method publicly through financial intermediaries. This allows large banks the option to use a Value-at-Risk measure to set the capital reserves necessary to cover their market-risk exposure. Regulators expect social benefits assuming that Value-at-Risk – based risk management will reduce the likelihood of large-scale financial failures.

The existing Value-at-Risk – related academic literature focuses mainly on measuring Value-at-Risk from different estimation methods to various calculation models. The first classical works in Value-at-Risk methodology distinguish mainly three basic estimation concepts: historical, Monte-Carlo and scenario simulations. Duffie and Pan (1997) used a relatively standard Monte-Carlo procedure, under which returns are simulated, and delta-gamma approximations to derivate prices are used to estimate marks to market for each scenario. Repeated simulation is used to estimate a confidence interval on losses that is typically known as Value-at-Risk. Cardenas, Fruchard, Picron Reyes, Walters, Yang (1999) and Rouvinez (1997) developed a recent Monte-Carlo Value-at-Risk estimation methodology with consideration of default risk. They were able to show how to replace the simulation step with an analytical method based on the characteristic function of the delta-gamma approximation of the change in portfolio value. With scenario simulation Jamshidian and Zhu (1997) break the link between the number of Monte Carlo draws and the number of portfolio re-pricings by approximating the distributions of changes in the factor rather than by approximating the portfolio value. As a further step, Abken (2000) suggests the use of principal components analysis to reduce the number of factors. Each risk factor is then assumed to take only a small number of distinct values, leading to a small or, at least, a manageable number of possible scenarios, each corresponding to a portfolio value that needs to be calculated only once. According to Bruderer and Hummler (1997), practical experience shows that Monte-Carlo simulation is then done by sampling among these scenarios, leading to a great reduction in the number of portfolio revaluations required.

In today's research and financial literature there is an enormous quantity of different Value-at-Risk calculation models, from the basic Value-at-Risk model to the dynamic Value-at-Risk model.[1] In spite of the advantages of these calculation models, Value-at-Risk is usually a statically calculated risk measurement. This is why we will try to estimate dynamic Value-at-Risk in our research work. The effect of dynamic strategies for large portfolios with a lot of financial titles can only be evaluated by Monte-Carlo methods. Nevertheless we examine all three estimation methods for the Swiss banking system and its portfolios.

2.2. Objectives and research questions

The main purpose of this essay is to present, develop and empirically test different Value-at-Risk estimation models. To this end, we first give an overview of the use of risk management and risk control and monitoring systems in practice with examples from the Swiss banking system. We clarify how banks explain the Value-at-Risk concept for interim goals to shareholders and clients.

[1] See Chapter 6.

Secondly, this paper addresses questions related to estimation methods of risk calculation. This means the wide implementation of Value-at-Risk will be observed in this study, and its advantages and disadvantages will be analysed. We not only briefly give a historical overview of evaluation of the Value-at-Risk concept but also verify whether these risk measurements, with their unique structure and estimation models, have a future. For different portfolio types we look at the pros and cons the use of estimation methods and apply Value-at-Risk – based risk management models. The main focus of this study is on empirical research of various Value-at-Risk evaluation models. The research explores and analyses in detail the building of risk control systems and the Value-at-Risk concept, both based on the core ideas of risk evaluation technique.

Thirdly, as the most popular asymmetric or down-side risk measure, Value-at-Risk offers a degree of practical interpretability often lacking in other risk measures such as volatility, liquidity, and data correlation. Therefore, as the next step we will research the connection between Value-at-risk and the other risk factors. This thesis provides a unifying approach to the valuation of all portfolios and almost all derivatives. A special topic of this study is to make clear whether we are speaking about Value-at-Risk or Profit-at-Risk. For these reasons we observe the current situation in modern discussions regarding Value-at-Risk and alternative risk measures. The risk of a portfolio depends not only on the composition of the portfolio, but on the objectives of the institution that holds the portfolio. Thus, two different institutions holding the identical portfolio, might view the risk differently. Value-at-Risk has already been adopted as a measure of portfolio risk. There is no doubt this is why regulators and industry and financial groups are now advocating the use of Value-at-Risk.

The opportunities that Value-at-Risk brings to the risk management system are great and obviously beneficial to both sides. It is used for trade limits and capital allocations. While Value-at-Risk is an extremely valuable tool and represents a significant step beyond previous measures, it cannot be applied in all situations equally. But nevertheless the shortcoming of modern Value-at-Risk research is that risk is statically calculated. Value-at-Risk provides quantitative and synthetic measures of risk. Certainly these advantages permit one to take into account various kinds of cross-dependence between asset returns, fat-tail and non-normality effects. Although there are some papers and research studies about dynamic and sensitivity analysis of Value-at-Risk, it is now early to define dynamic Value-at-Risk as further developed risk measurement tool.

The objective of this study is to address the above-mentioned research questions by using a wide set of daily data and to analyse the application of Value-at-Risk in portfolio management with special concentration on the dynamic Value-at-Risk model. In our paper we try to answer the following initial questions:

- What does the Value-at-Risk concept deal with?
- Which estimation methods and models are more optimal for Value-at-Risk measurements?
- Why do we need and how can we apply the Value-at-Risk concept in portfolio management?
- What advantages and disadvantages can appear and have a crucial influence in banking daily routine by using the Value-at-Risk models?
- How can we measure portfolio Value-at-Risk dynamically?

3. The Economic Importance of Value-at-Risk

In banking system risks are of various natures, which have from time to time not really received a complete explanation in the literature. The differences of risk types can be described in the following way (see Figure 1). There are primarily four big risk arts: operative, financial, trade, and strategic risks in the banking system.

Operative risk includes legal risks as well as technical risks. Trade risk appears through the changing demand of bank instruments und competition issues. Under strategic risks the danger of a whole or partial failure of the financial system is to be understood. In this work we speak more about financial risk, which consists of credit risk, market risk and liquidity risk. Credit risks are to be led back on credit standing changes or the inability of the opposition to pay. The nature of liquidity risk comes from re-financing and possible delays in payment. But we are only interested in market risks for now.

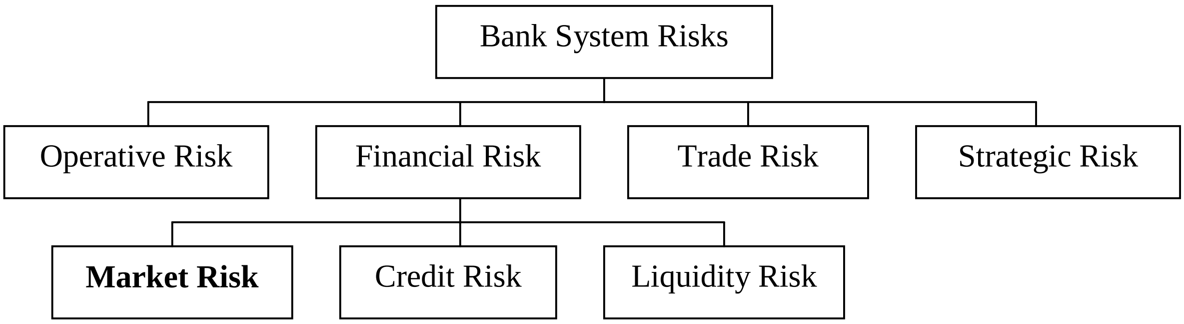

Figure 1. Risks in the banking system.

The most widely used tool to measure, gear and control market risk is Value-at-Risk. The financial and economic world really needs this measure as it serves for a number of purposes. Value-at-Risk builds an information report. It can be used to apprise senior management of the risk run by trading and investment operations, this also means a clear communication of the financial risks to the shareholders of corporations in non-technical terms. Thus, Value-at-Risk helps speed up the current trend towards better disclosure based on the mark-to-market reporting.

The second point is resource allocation. Nowadays, especially in private banks and insurance companies, Value-at-Risk is used to set position limits for traders and to decide how to allocate limited capital resources. The advantages of Value-at-Risk allow creation of a common denominator with which to compare risky activities in diverse markets. The total risk of the entrepreneur, firm, bank, and corporation can also be decomposed into “incremental” Value-at-Risk to uncover positions contributing most to total risk.

On the other hand, Value-at-Risk can be used to adjust the performance of risk. Performance evaluation is essential in a trading environment, where traders have a natural tendency to take on extra risk. Risk-capital charges based on Value-at-Risk measures provide corrected incentives to traders.

We have already mentioned that the Value-at-Risk concept is being adopted as a risk measure by financial institutions, regulators, non-financial corporations and asset managers[2].

[2] Institutions that deal with numerous sources of financial risk and complicated instruments are now implementing centralised risk management systems. The prudential regulation of financial institutions

In the end, the greatest benefit of Value-at-Risk probably lies in the imposition of a structured methodology for critically thinking about risk. Through the process of computing their Value-at-Risk, institutions are forced to confront their exposure to financial risk and set up an independent risk management function supervising the front and the back offices. Indeed, reasonable use of Value-at-Risk may have avoided many of the financial and economic disasters experienced over the past couple of years. There is no doubt that Value-at-Risk is here to stay.

4. Value-at-Risk Calculations in a Praxis

4.1. The Estimations by one Swiss Private Bank

Value-at-Risk is the most famous risk characteristic tool in the practice of the risk estimates. In our initial empirical research we tried to connect the theory of risk management and the practice aspects of risk analysis. There are a few different appreciation methods of the risk calculation. We use an estimation method from one Swiss private bank. We analyse the connection between the Value-at-Risk, possible risk factors and trade limits. The special point of discussion is the application of the Value-at-Risk concept for clients' portfolios. The first research question is connected with the possible influence of non-linear products on the Value-at-Risk. Then we discuss trade limits and dynamic strategy to measure the Value-at-Risk.

The practice of the bank-operational risk management shows different main key points and drafts in regard to the risk calculations. In this study the peculiarities in the risk policy of a Swiss private bank and in the risk estimates for the clients' portfolios are shown. The oldest bank in Switzerland, Wegelin & Co., is taken as an interesting example. According to their three estimation techniques, and namely historical simulation, stress test and scenario simulation. This Swiss bank has already been calculating portfolio Value-at-Risk for all their clients for the past 3 years.

The risk policy of Wegelin & Co. is based on the risk responsibility, risk management and risk control. One can define three competence steps. On the first level the management remits the measures to the risk limitation and formulates risk policy in particular. Then the risks are aggregated from the partial books and current tax structures. Finally the bank's management and the risk board define the responsible groups and delegate the risk supervision to them. The responsibility of risk estimations of customer portfolios belongs to the daily work of every investment consultant. However, simply with derivative instruments, like structured products, it is difficult to calculate Value-at-Risk at first sight for non-linearity reasons. In these reports, the risks of the investments are measured and presented in a transparent manner. This is especially important for structured products, which can behave either like stocks or like bonds, depending on the product design and the price movement of the underlying securities. The next figure shows us the general structure of the risk control system of this private bank.

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requires the maintenance of minimum levels of capital as reserves against financial risk. Centralised risk management is useful to any corporation that has exposure to financial risk. Value-at-Risk allows such corporations to uncover their exposure to financial risk, which is the first step toward an informed hedging policy. Moreover, institutional investors are now turning to Value-at-Risk to better control financial risks.

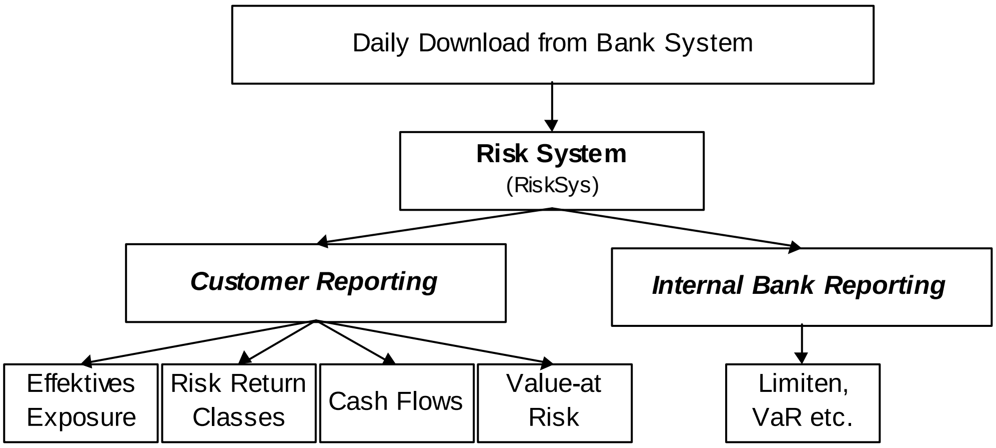


Figure 2. Wegelin Risk Control System (RiskSys)

As the first step the effective exposure was supplied. The breakdown of the total assets by the basic investment categories (equities, interest bearing investments and tangible assets) and by currencies are shown. Derivative instruments are broken down in proportion to their actual economic characteristics into the respective investment categories. The report by risk/return classes permits the client advisor to monitor the allocation in accordance with the Wegelin investment process on a daily basis (breakdown of the portfolio into four risk/return classes: blue, green, yellow and red). Within the individual risk/return classes, the investments are further divided into industry sectors and geographic markets (risk/return classes yellow) or duration and currencies (risk/return classes green).

The Value-at-Risk presentation shows the potential loss for the portfolio under different scenarios. We define the Value-at-Risk as the highest possible loss that can enter within a certain time with a certain trust level. As a basis formula the following equation applies in addition:

$$VaR = \alpha \cdot \sigma \cdot \sqrt{T} ,$$

where α is the confidence level (α is the distance of the means measures in number of standard deviations: for example, 1.65 corresponds to 95 %-confidence level), σ is the portfolio volatility, which is measured as a standard deviation of the yields and T is time period.

We already know that the Value-at-Risk grows proportionally with the time. In this case the Portfolio Value-at-Risk is the maximum loss that the portfolio experiences under the acceptances of the scenario. Mathematically it is formulated as the minimum of the differences between the portfolio value and the simulated scenario value:

$$VaR = min\{D1, D2, ...\},$$

$$D1 = ScenarioValue1 - PortfolioValue .$$

In the first research steps we looked at the Value-at-Risk estimation methods at a private bank to clarify our questions. What importance do the limits on the risk estimation of portfolios have? How big is the range of application of the Value-at-Risk for the customer reporting of a private bank. And which factors have the most influence on the risk calculation?

4.2. Wegelin Value-at-Risk Scenarios

As the effective exposures reports only allow the estimation with small market movements of a possible loss, still the Value-at-Risk calculations were implemented in addition. The Value-at-Risk concept scenarios are defined to calculate the changes in market risk factors and the potential losses, which would result with the occurrence of the scenarios. The following risks are defined in RiskSys: share quotations, the volatility of the share quotations, interests, exchange rates, as well as price of raw materials. To calculate the changes in portfolio value, all financial instruments are estimated according to scenarios and risk factors. The difference between the current value of portfolios and the simulated value (based on the changed risk factors) proves the Value-at-Risk and the capital, which could get lost under the acceptance of the occurrence of the scenario for the hold period.

The following are the three scenarios used by Wegelin (see Figure 3):

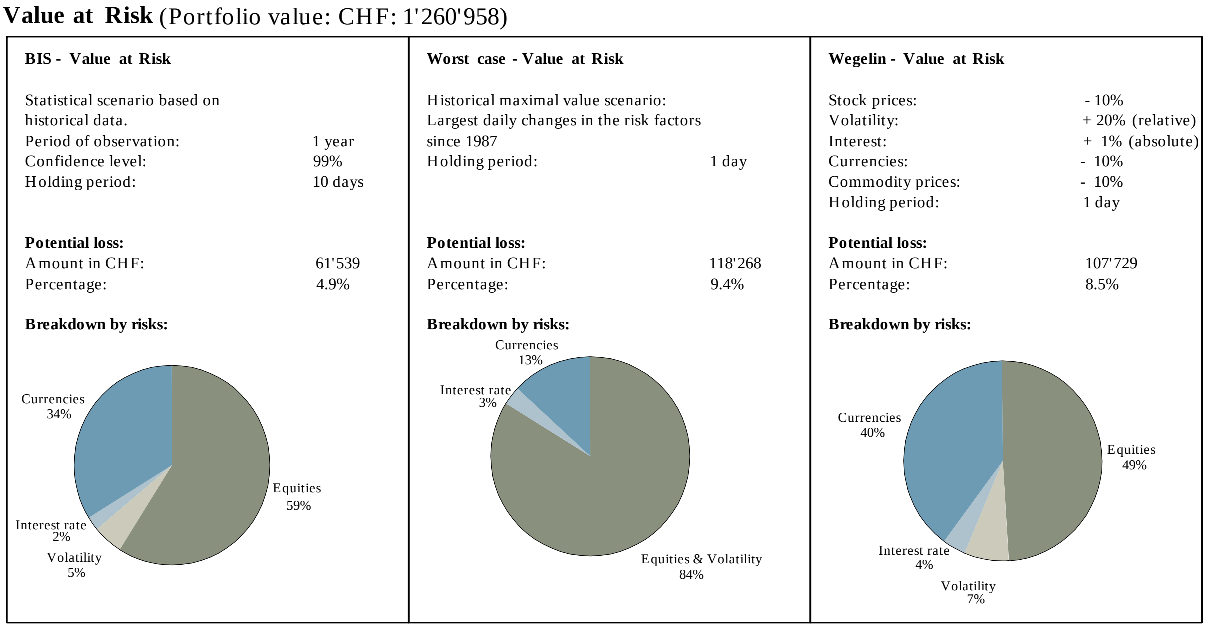


Figure 3. Wegelin Value-at-Risk.

The first scenario is the so-called BIZ scenario. It is a standard statistic scenario, where the risk factors for a certain observation period are analysed statistically. The information serves as an input for the definition of the scenarios. The effects are considered by two scenarios in each case per risk factor (one with a positive change of the risk factor, and one with a negative change), so that no additional acceptances regarding the type of the portfolios (long position or short) must be fixed. The Bank for International Settlements (Bank für Internationalen Zahlungsausgleich - BIZ) recommends the following values of the scenario: at least 1 year as an observation period, 10 days as a holding period, and 99% confidence level.

For the stress (or worst case) scenario, the shifts in the risk factors are coming from the historically observed extreme value changes of the risk factors. They are given by the historical maximum or minimum of the changes. In addition, the extreme values can have appeared at different moments. The biggest interest change observed since 1987 defined the shift in the interest, the biggest change appeared in the stock markets (observe, for example, in 1987) defined the shift in the shares for the worst case scenario. The aim of the stress scenario is to show worth changes of portfolio values, if the most extreme changes observed in the past of the risk factors would all occur one day at the same moment.

The last model is Event scenario or Wegelin scenario. This scenario is very simple to understand. It is a sort of sensitivity analysis of the portfolio values. It shows exactly how defined changes of the risk factors would have an effect on the current portfolio value. In customer communications, this scenario is used most.

The bank Wegelin & Co. makes it possible to calculate the Value-at-Risk for every customer from institutional to private as well. However, the interest in Value-at-Risk reports is very much selective and is request by approximately 5-10% of all customers. Most customers who really use Value-at-Risk reporting are satisfied with the Value-at-Risk representation that shows potential losses to them. The investment consultant compares by the calculation of the Value-at-Risk, the risk to the possible yield by performance estimation of portfolios. The Value-at-Risk plays a special role in the estimation of the structured finance products which are defined as combinations by two or several arrangements or derivatives. Of course every structured product gets its own risk profile, which changes according to the specific risks of the separate underlying instruments. Therefore, special risk measurement is needed for structured products, like the Value-at-Risk to be able to measure and explain the risks adequately as well by complex finance instruments. Certainly the Value-at-Risk concept also helps the bank management a lot. With its assistance, the whole market risk of the bank is defined and trade limits are set. The Value-at-Risk makes it simple for the traders to determine where the brief capital resources should be used. And customers can see for themselves their specific report risk.

The chief range of Value-at-Risk application remains through conversations with customers. The example of Wegelin & Co. shows the advantages of the Value-at-Risk concept for the customers and for the risk estimation of finance instruments like structured products. The other development of the Value-at-Risk method in this Swiss bank is the dynamic approach. This refers to the creation of a risk-adjusted performance measurement and the better clarification going in the customers' and investment consultants' direction with the application of the trade and portfolio limits.

5. Empirical Research

5.1. Value-at-Risk and Limitation

Our empirical study is based on the estimation of the Value-at-Risk for approximately 1500 Portfolios. For the past 3 years, the Value-at-Risk has been calculated by Wegelin & Co. for their own customers. Since this time there have been more than 1600 portfolios altogether for which the Value-at-Risk can be counted. But we have deliberately expelled some portfolios from our analysis. We do not observe such portfolios which have only currency risk or small weight in the whole capital structure. For more than 1500 objects we checked whether the risk estimation was correct for the customer portfolios during the observation period or not.

For every portfolio, the Value-at-Risk defines maximally possible potential losses under the concerning presuppositions. Therefore the daily limits are thusly fixed, that the daily transactions are not crossed. For the general market risk, the year limits are given by the management group at the beginning of the financial year in the form of the Value-at-Risk with a 95% confidence level. These year limits are converted into a form of the dynamic limits for the holding period relevant to the trade a day. The conversion for the one-day holding period applied in the RiskSys and 99% confidence level is based on using the Square-T-formula. The use of it for non-linear positions is problematic. The effect, that the Square-T-formula has on the observance of the year limits, was not up to now examined for such positions. However, the loss-limiting settlement of the daily trade results is a very careful procedure. Therefore, the real year losses cross the year limits in less than 5% of the cases.

During 250 trade days the daily limit (*TL*) arises from the year limit (*JL*) in accordance with the following formula:

$$TL = \frac{JL}{\sqrt{250}} \frac{2.326}{1.645}$$

The daily profits and losses (ΔV_t) are added from the beginning of the year ($\Delta \sum V_t$) and supplied to the original year limit (JL_{Start}). But the accordingly calculated new year limit (*JL*) may not exceed the fixed original year limit (JL_{Start}). The new year limit, which is adapted daily on account of the trade results, is converted in accordance with the above formula on a daily limit. For the day limit arises:

$$TL = \begin{cases} \frac{JL_{Start}}{\sqrt{250}} \frac{2.326}{1.645} & ,if \quad \sum \Delta V_t \geq 0 \\ \frac{JL_{Start} + \sum \Delta V_t}{\sqrt{250}} \frac{2.326}{1.645} & ,if \quad \sum \Delta V_t < 0 \end{cases}$$

One observes the changes for the scenario with the definition of the money/exposure limits and implementation in an amount of money back. As the Value-at-Risk can also be determined for other risks than the market risks, the Value-at-Risk limit system is suitable fundamentally to the control of all bank risks. This department makes clear the structure of a Value-at-Risk limit system for the trade area of a bank (see Figure 4).

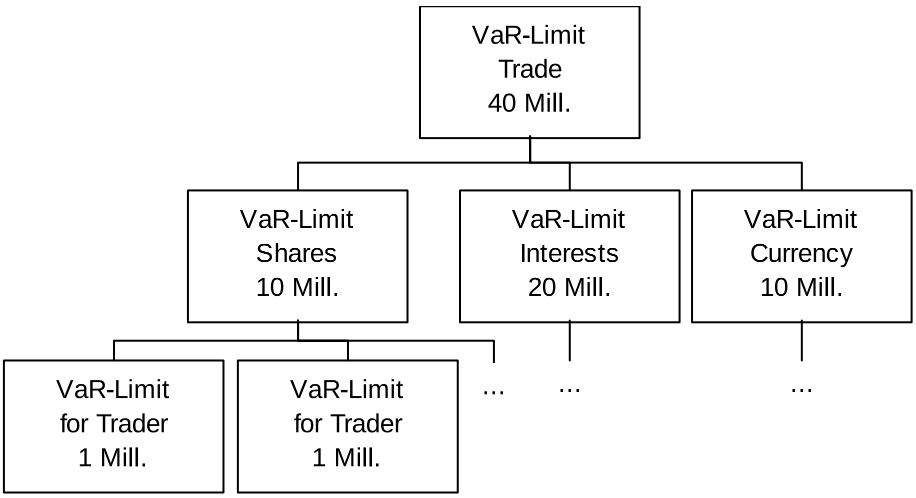


Figure 4. Value-at-Risk Limits System.

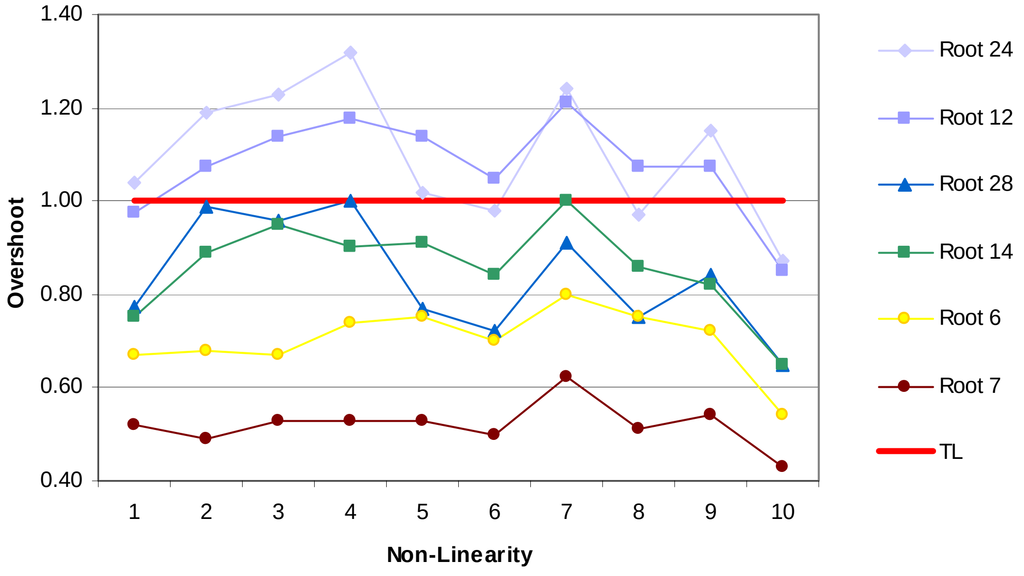
For the trade area, for example, a limit of 40 million is available, which is distributed by consideration of the correlation between the risk categories to the areas of shares (10 million), interests (20 million) and currencies (10 million). A year Value-at-Risk limit is available to every trader from 1 million.

5.2. The First Empirical Results

In our empirical analysis, we have a data row with approximately 1500 portfolios. As was already mentioned, one considers the 2-year-old time interval from the 31st of July 1999 to the 30th of November 2001. As the distribution of the data unfortunately is discrete and is not normal, the historical Value-at-Risk is calculated according to the standard scenario. At the private bank Wegelin & Co. one hopes that the real losses might not be largely more than the fixed Value-at-Risk for every Portfolio with a probability of 99%. In reality it is so only with about 28% of all the portfolios. But nevertheless we can state that all the analysed portfolios have the correct Value-at-Risk estimation in 90% of cases and the daily limit were crossed only in less than 5% of all cases during the observation period.

In the next step we calculated the historical Value-at-Risk and interpreted it from the side of the non-linearity of the instruments and from the key point of the daily limit adaptation and autocorrelation between data. We looked at whether there was any connection between the interest of the non-linear instruments in the portfolios and the scaling rank in the Square-T-formula. For us it is interesting to not how important the dynamic changes of the Value-at-Risk is and how the data are connected.

As in the bank trading area, trade contracts and bank deals are usually closed in the short-term period, it is to be cleared, how the year limits can be converted in a daily, weekly and / or month limit and which role the Square-T-formula have for this calculation (see Figure 5). It is significant to find the correct connection between the scaling rank and the number of the non-linear portfolio. That is why we calculated Value-at-Risk for different *T*s.



Figures 5. Positive Yield Correlation

The results show us a clear positive autocorrelation. The higher lying curves for 2 and 4 weeks point to a positive correlation of the yields, which would also express themselves in a rising Variance Ratio. There are no essential differences between the portfolios. It depends more on the data rows and less on the composition of the portfolios. The temporal scaling rank is almost the same for all of them. This means that, we must consider data rows instead of the instruments with an alternative to the root t rule.

Coming back to the problem of limits re-valuation, we just built the example portfolios with one year limit in 10 million CHF. In our model we made the Value-at-risk simulation with 10000 simulations according to Monte-Carlo models for daily price changes during one year with a suggestion for 250 active trade days a year. We made the proposal that the Value-at-Risk limits are absolutely used, price changes are normally distributed with Brown's geometric fluctuation, the drift is 10% and volatility is 23%. We simulated portfolio value according to two strategies (i) with and (ii) without limits re-valuation (Figure 6).

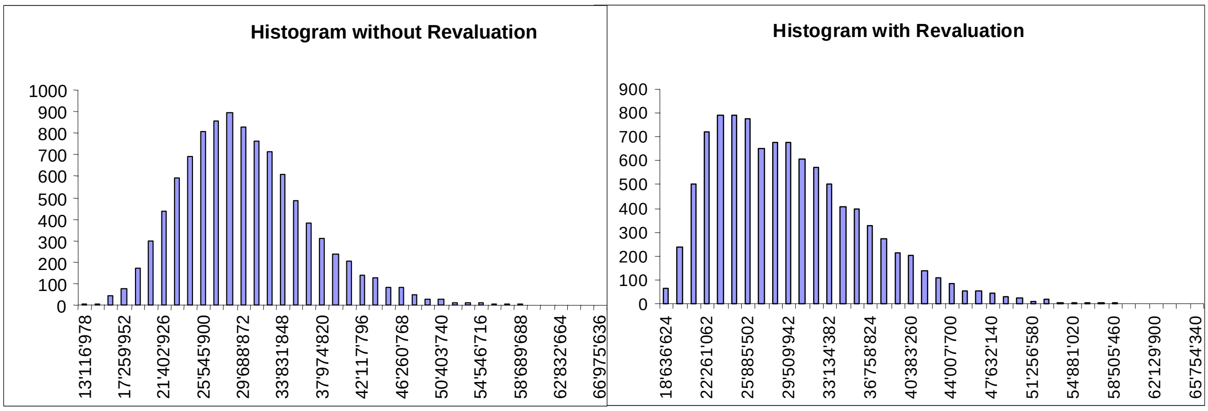


Figure 6. Final Portfolio Value

As we can see in the case of the Value-at-Risk re-valuation, the distribution is bimodal. It is also interesting to research the statistic results for different Bid-Ask-Spread and various distribution models. We then have the following results:

Portfolio with normal distribution:

Initial Value	26'430'203						
	Without Adaptation	BA 0%	BA 0.1%	BA 0.5%	BA 1%	BA 2%	BA 5%
Min	11'735'987	17'428'477	17'448'053	17'445'583	17'202'505	16'892'767	16'216'149
Q1%	16'879'068	18'975'762	19'144'414	18'903'366	18'775'658	18'426'934	17'341'498
Q5%	19'644'572	20'399'433	20'532'504	20'248'878	20'126'737	19'781'363	18'593'425
Mean	29'265'757	28'957'366	29'063'361	28'956'035	28'733'028	28'562'492	28'012'655
Median	28'496'704	27'921'665	28'022'230	27'857'234	27'641'117	27'510'780	27'005'116
Max	66'975'637	65'754'341	66'964'020	72'192'457	71'716'391	76'521'713	61'977'890
Q95%	41'713'937	41'202'052	41'618'212	41'739'442	41'302'768	41'253'481	41'446'201
VaR5%	9'621'185	8'557'932	8'530'857	8'707'157	8'606'291	8'781'129	9'419'230

For the case with fat tails we get the next portfolio statistic:

Initial Value	26'430'203						
	Without Adaptation	BA 0%	BA 0.1%	BA 0.5%	BA 1%	BA 2%	BA 5%
Min	12'722'286	13'454'700	12'374'622	16'533'836	16'501'309	11'895'311	15'711'993
Q1%	16'195'636	18'797'043	18'850'165	18'594'197	18'488'085	18'195'954	17'190'825
Q5%	19'104'799	20'191'279	20'144'042	19'972'284	19'851'941	19'460'444	18'421'693
Mean	29'217'308	29'082'948	29'039'671	28'904'012	28'889'634	28'601'573	27'908'764
Median	28'400'152	27'869'712	27'977'123	27'651'700	27'706'101	27'504'761	26'668'891
Max	68'584'156	72'269'242	70'394'007	71'464'684	72'078'490	73'657'798	70'807'413
Q95%	42'361'242	42'149'674	41'796'426	42'395'778	42'329'820	41'665'998	41'837'338
VaR5%	10'112'509	8'891'669	8'895'630	8'931'728	9'037'693	9'141'129	9'487'070

These results show us quite obviously that for the assumption regarding normal distribution we have a situation when in the worst case the limits are fully used. For the fat tails study the Value-at-Risk limit can be significantly more implied (Figure 7).

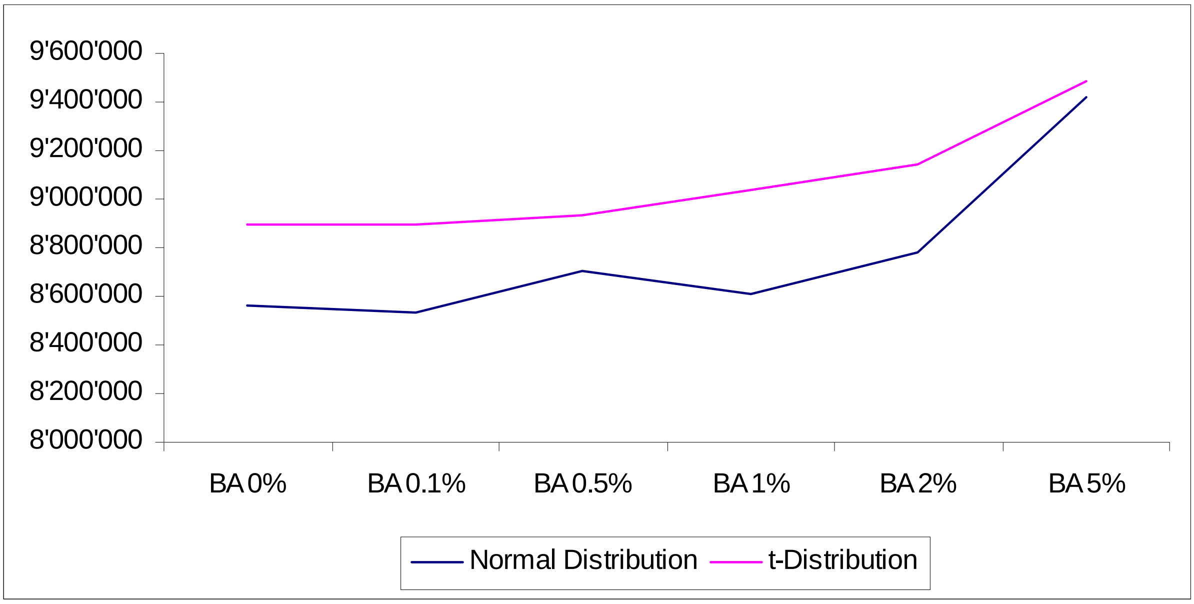


Figure 7: Value-at-Risk for different Bid-Ask-Spreads and various Distribution

5.3. Conclusion

Our first empirical results show that Value-at-Risk can be used to quantify the market risks in different portfolios. The risk management models that have been developed initially for this purpose estimate current and future market value of portfolios and can preserve the non-linear effects of some obtained financial instruments such as options and structured products. The Value-at-Risk models that are obtained integrate the most relevant risk factors: interest rate risk, liquidity risk and prepayment risk. Moreover, the Value-at-Risk also accounts for the risk factors' dependencies as well.

Through the calculation of a Portfolio Value-at-Risk there is first the problem of how the different risk masses can be used on the same time horizon. So it usually will be necessary to scale the Value-at-Risk for market risks on longer time horizons. This means that it needs to define the general calculations and estimations rules. As our results show, there are some other problems here, such as the abnormality of empirical yields around longer horizons according to the Square-T-formula or the various portfolio quantities of the bank bench. One of the main focuses of this paper deal with the second problem. We tried to make clear the connection between the Value-at-Risk and different risk factors. The special results of the empirical study force us to look at the γ - scaling by the non-linearity of the instruments.

Nevertheless one can already determine the next research steps. It is clear that the non-linear instruments are not the single risk factor. Other factors like parameter instability, and correlation should be considered. For better clarification of the customer portfolios we must also calculate the risk adjusted performance ratio. Here in the first line one should speak more about the dynamic approach to the Value-at-Risk estimation. This means the analysis of Value-at-Risk changes under the consideration of the daily adaptation of trade limits. The variables portfolio weight parameters explicitly have been considered for market risk simulation to build the internal trade rules for setting so the dynamic Value-at-Risk on longer time horizons.

If we follow the strategy to adapt portfolios according to defined limit rules we have bimodal distribution of portfolio value changes. The other interesting point that we could see is that by portfolio limits adaptation the Value-at-Risk is about 10 % smaller in the case where we do not assume Bid-Ask-Spreads. With higher spreads this advantage disappears more and more. Finally, the worst case is close to the defined portfolio limits in the situation with portfolio adaptations. But it does not concern heavy tails.

In our empirical case study, we used both of the most applied Value-at-Risk estimation methods: historical as well as Monte-Carlo simulations. The essence of Value-at-Risk is the prediction of the highest expected loss for a given portfolio. The Value-at-Risk techniques estimate losses by approximating low quantiles in the portfolio distribution. The delta methods are based on the normal assumption for the distribution of financial returns. The empirical data exhibits fat tails and excess kurtosis. The historical method does not impose the distributional assumptions but it is reliable in estimating low quantiles with a small number of observations of tails. The performance of the Monte-Carlo method depends on the quality of distributional assumptions on underlying risk factors.

The existing methods do not provide satisfactory evaluation of Value-at-Risk. The proposed improvements still lack a convincing unified technique capturing the observed phenomena in financial data. We demonstrated examples of the Value-at-Risk estimations under different assumptions according to the model parameters. This methodology was recently adopted by one Swiss private bank that uses such models not only to capture the risk profile of the customer portfolios, but also to asses the efficiency of the funding allocated to the particular market risks in these portfolios. In this way, the Value-at-Risk concept can definitely be used to determine the optimal dynamic strategy and minimise portfolio costs, given a level of risk that is acceptable.

6. Dynamic Strategy of Value-at-Risk Estimation

Value-at-Risk estimates often move through time. As we have shown, generally be adjusting linear Value-at-Risk measures as a square root of time. However, in order to properly model risk stemming from exotic option portfolios as a function of time, it is essential to capture multiple-step rate paths. By generating such paths, it is possible properly to capture the reset and exercise effects and the risk of crossing potential option barrier levels when calculating Value-at-Risk. Once multi-step risk has been calculated, it is necessary to provide risk managers with proper hedging tools to ensure that certain risk factors are neutralised when Value-at-Risk measures adjust through time. It is necessary to create a portfolio environment that accurately reflects actual steps that would be executed by traders to minimise risk. In initial Monte Carlo implementation, an n-step rate path will be introduced and risk managers will be able to ensure delta neutrality by automatically applying hedges with one or two liquid instruments such as the appropriate near and far futures contracts. In subsequent implementations, hedges to neutralise portfolio sensitivities (both gamma and vega) risk will be employed and the number of available hedging vehicles will be expanded.

The Value-at-Risk measure represents an estimate of the amount by which an institution's position in a risk category could decline due to general market movements during a given holding period. Of course, the Value-at-Risk is one rude approximation to measure market risk. To put it even more plainly, Value-at-Risk is a percentile of the profit and loss distribution of a portfolio over a specified horizon. The distribution results are then used in the Monte-Carlo simulation process by limiting the application of randomly generated rate path to those that are statistically relevant given the portfolio anticipated risk profiles. To make the Value-at-Risk estimation more precise, we need to use dynamic strategies to measure the Value-at-Risk.

The dynamic strategy can be most appropriately illustrated by applying it to real situation in portfolio management. The market conditions are changing every day. That is why, we must adapt the Value-at-Risk estimation to the daily, weekly or monthly fluctuations. Moreover, if we suggest trading limits and portfolio adaptation to the maximal possible losses of a portfolio, which are defined by asset managers, we can see, the Value-at-Risk measures are considerably different. The problem of dynamic Value-at-Risk deals with the questions, how one can define the general trading rules and build a single adaptation scheme for risk estimations.

Every bank or financial institute use their own Value-at-Risk system. In real life a trader is most probably a vega trader and he positions his trading book to his future expectations. He will either gamma trade in expectation of higher realised volatilities, or sell options as the market maker of warrants in expectation of lower realised volatilities. Pro-cyclical trading strategies, which buy stocks as they increase in value and sell stocks as they decrease in value, are defined as convex strategies. Counter-cyclical trading strategies, which sell stocks as they increase in value and buy stocks as they decrease in value, are called concave strategies.[3] Portfolio mangers try to optimise their risk / return profile by properly adapting these strategies. They can lead to the same conclusions as market makers in

[3] Kubli (2001)

forecasting future stock prices movements and how they should position themselves to profit from these expected market conditions.

Generally speaking, the problems of a dynamic Value-at-Risk measurement are difficult to solve, but very interesting and still actual in modern risk research. In our paper we would like to define the general trading limits and adaptation rules to use them in estimation model. Daily markets movements and portfolio adaptation according determined limits and rules are crucial factors in the dynamic Value-at-Risk appreciation.

7. Outlook

In this paper the use of the Value-at-Risk concept in portfolio management with examples from Swiss banking system is undertaken. We looked trough the problematic of risk measurement in portfolio management to define the initial research questions, objectives and purposes of our study. From the point of current researches in modern financial literature we discussed the economic importance of Value-at-Risk concept in portfolio management. We analysed its evaluation and methodological development of this concept. The significant part of our thesis is theoretical basis of the Value-at-Risk. We analysed the different estimation methods as well as existing Value-at-Risk models in details. The special topic of modern discussion in finance is extreme value theory and implementation of extreme value distribution in risk measurement. We put our attention on these related questions from the point of the Value-at-Risk concept.

Wegelin Value-at-Risk model, which are based on three main estimation methods: historical simulation, Monte-Carlo method and scenario simulations with the worst case study, is implemented in the first empirical research. We compare these estimation approaches by the risk measurements in a praxis. Their advantages and disadvantages are represented here. As a result of our discussion we introduced the trading limits system in the Value-at-Risk appreciation. The influence of different risk factors such as non-linearity, liquidity, distribution assumption etc. are also analysed. As a new modern measurement technique dynamic strategy is suggested for further implementation in the Value-at-Risk estimations.

There are a couple of different directions for our research to go, such as other process assumptions, various portfolio structures, liquidity and correlation analysis, other estimation methods, and different limit rules and their adaptation schemes. Before having them analysed with the help of analytical models and risk management theory, we began our thesis with the research of a simple Value-at-Risk model in order to fulfil the need of an applicable dynamic approach to the risk measurement. For the next research steps we would suggest following points.

First of all, our empirical analysis has been performed under some restrictive conditions, namely normally distributed returns and constant portfolio allocations. These assumptions can be weakened. For instance, we can introduce non-parametric Markov models for returns, allowing for non-linear dynamics and compute the corresponding conditional Value-at-Risk together with their derivatives. The suggested effect of a dynamic strategy on the Value-at-Risk can be evaluated, as earlier, by Monte-Carlo methods or the historical simulation approach.

Secondly, we observed that stable Value-at-Risk modelling outperforms the normal modelling at high values of the Value-at-Risk confidence level. For legitimate conclusions on the stable and dynamic Value-at-Risk methodology we need to test it on broader classes of risk factors and portfolios of assets of various complexity. That is why we need to research portfolios with different structures and various portfolio masses. Such an extension is currently under development.

Thirdly, risk limits have already been common tools for establishing and communicating clear boundaries around risk tolerances. However, these risk limits are often not developed to produce a relative risk unit consistency. Several system approaches provide functionality to establish limits and monitors that the portfolio does not exceed these limits. The simplest and most effective solution for a dynamic Value-at-Risk estimation is to research different limit models in praxis and to build a limit adaptation scheme for our further analysis. As the first steps in this direction, we are also going to define limit rules by different banks.

Along with a better understanding of the Value-at-Risk dependencies and connections between different risk factors we must also include liquidity aspects and analysis of correlation matrixes. Developing such a set of risk analytics is the next challenge in the evolution of the Value-at-Risk concept and our case study. The price development must be researched from the point of stochastic liquidity.

Finally, through a simulation methodology, we attempt to determine how each Value-at-Risk approach would have performed over a realistic range of portfolios containing different financial instruments over the same period. The question regarding the effective application of scenario simulations is still open here. It is not possible to suggest that the simulation result would be the same under various scenario assumptions.

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